Low order modeling and closed loop thermal regulation

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Workshop –Thermal Design & Dimensional Drift issues

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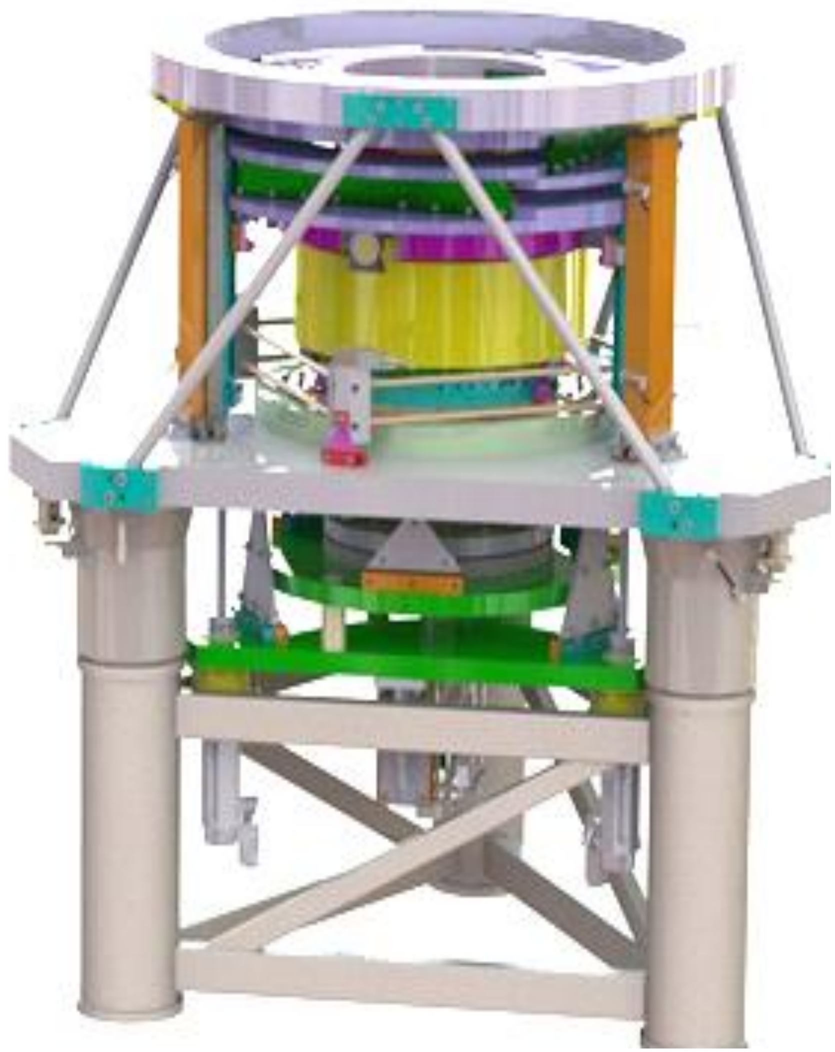
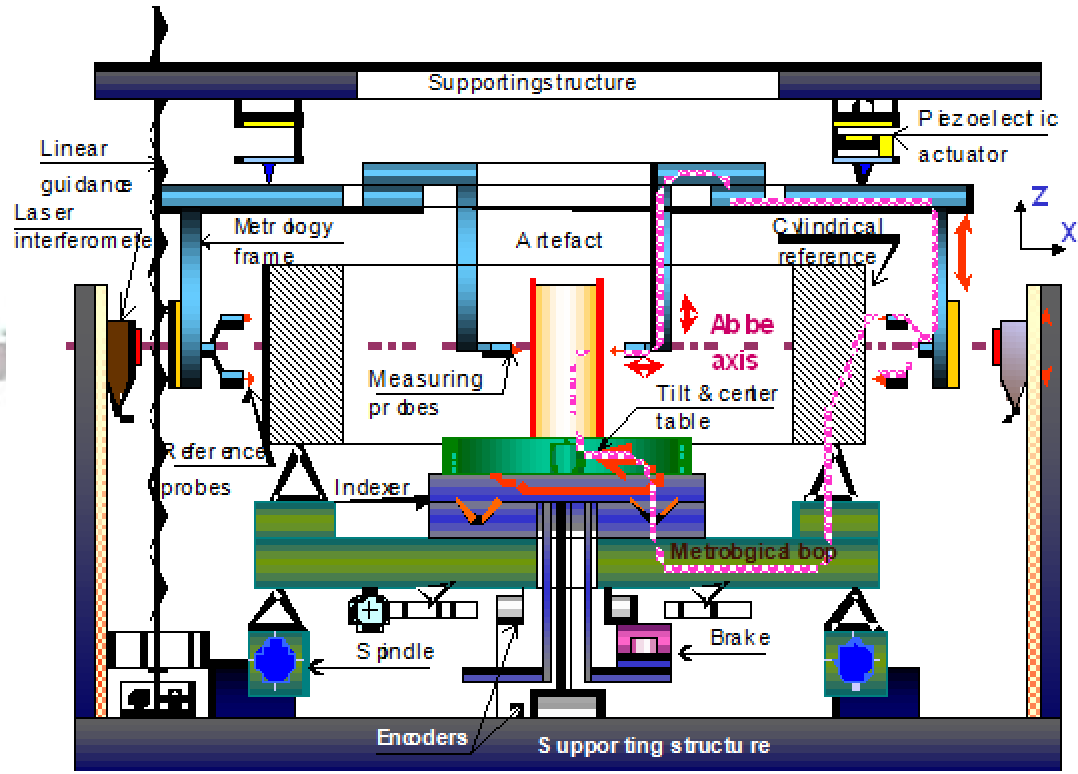
Context



New cylindricity measurement apparatus

Nanometric precision aimed



Principles applied :
- DMT
- ABBE

Symmetry insured

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Thermal disturbances

Thermal error sources

Internal heat sources
- Guiding systems
- Laser interferometers

External heat sources
- External temperature (Text)
- Operators

Temperature distribution in the machine (Uniform and non-uniform)

Thermal expansion (1mK induces 10nm)

Measurement errors

Thermal modeling

Thermal modeling :

Energy balance:

$$\vec{\nabla}.\left(k(M)\vec{\nabla}T(M,t)\right)+\sum_{j=1}^{n_Q}\left[\frac{P_j(t)}{V_j}\chi_j(M)\right]=\rho(M)C_p(M)\frac{\partial T}{\partial t}(M,t),$$

$$\forall\, M\in\Omega$$

Boundary conditions :

$$k(M)\vec{\nabla}T(M,t).\vec{n}=h(T_a(t)-T(M,t)),\forall M\in\Gamma$$

$$k(M)\vec{\nabla}T(M,t).\vec{n}=\frac{A_i(t)\xi_i(M)}{S_i}$$

Resolution of the heat transfer equation

Spatial discretization: - Finite elements
- Finite volumes
- Finite differences

State space representation:

$$\begin{cases}\dot{\boldsymbol{T}}=\boldsymbol{AT}(\boldsymbol{t})+\boldsymbol{BU}(\boldsymbol{t})\\ \boldsymbol{Y}(\boldsymbol{t})=\boldsymbol{CT}(\boldsymbol{t})\end{cases}$$

N ODE

Issues:

- Memory
- Large computation time
- Unsuitable for real-time control

Solution

- Find a model reproducing the behaviour of the system with a small number of differential equations (**model reduction**)

Modal Identification Method (MIM)

1. Definition of a suitable structure of the reduced model

Modal state-space representation

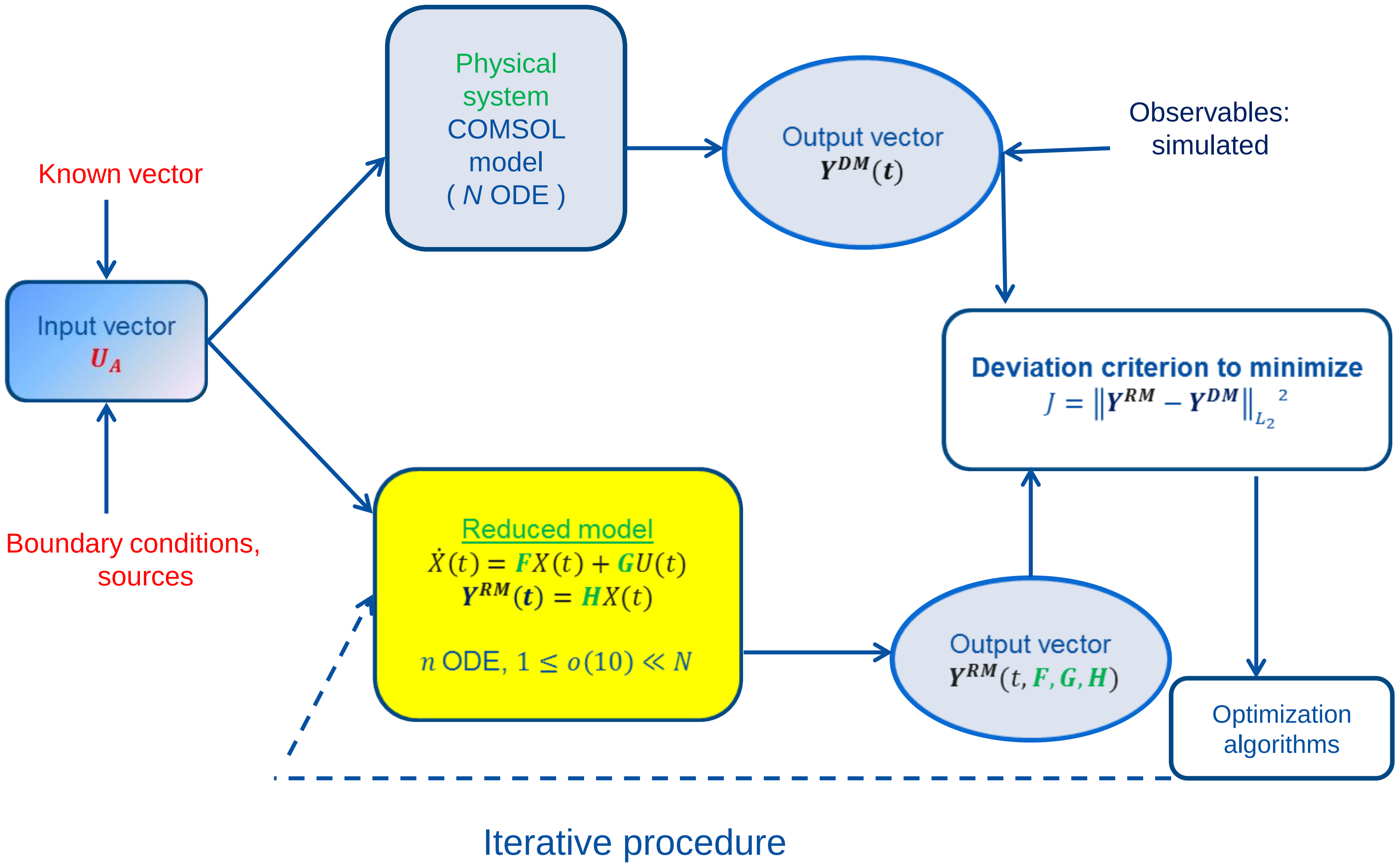
$$\begin{cases} \dot{X}(t) = \boldsymbol{F}X(t) + \boldsymbol{G}U(t) \\ \boldsymbol{Y}(\boldsymbol{t}) = \boldsymbol{H}X(t) \end{cases}$$ n ODE, n << N

2. Generation of numerical output data for a set of known input signals

3. Identification of the parameters of the reduced model (F,G,H) through optimization algorithms:

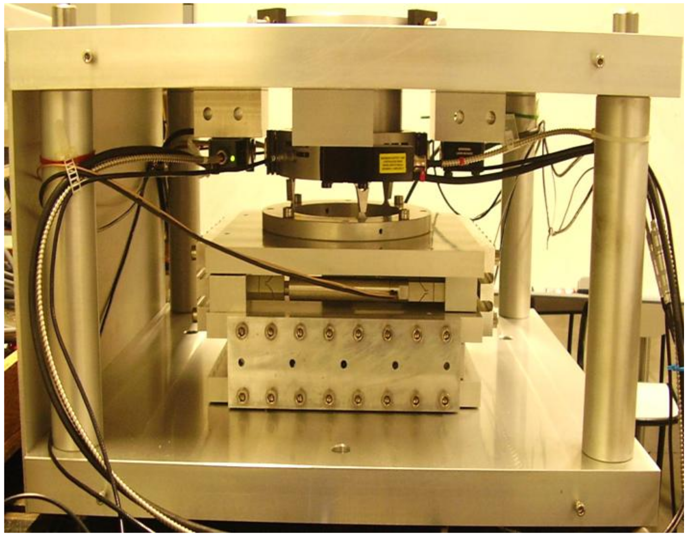
- Ordinary Linear Least Squares ($\boldsymbol{H}$)
- Particle Swarm Optimization ($\boldsymbol{F}$, $\boldsymbol{G}$)

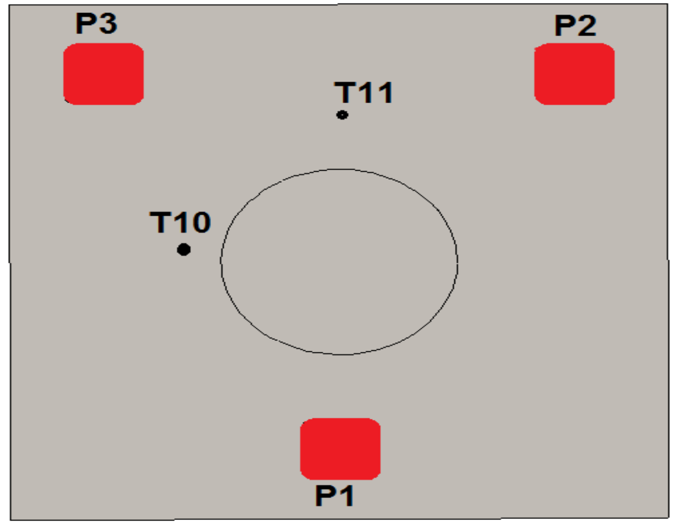
Model Identification Method



Model Identification Method scheme

Experimental device

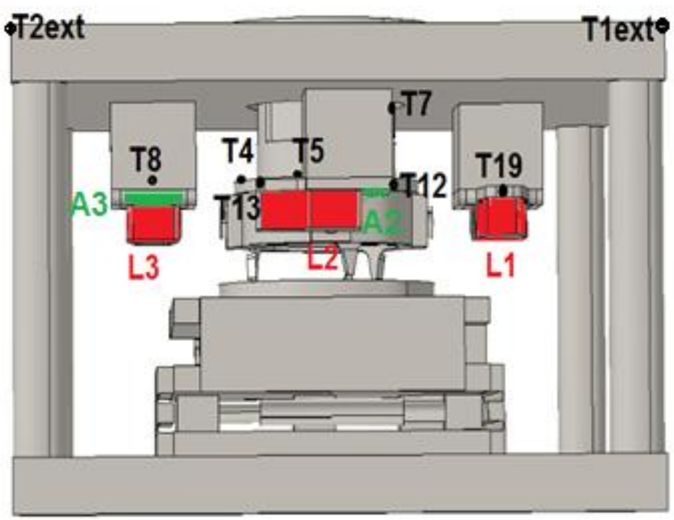


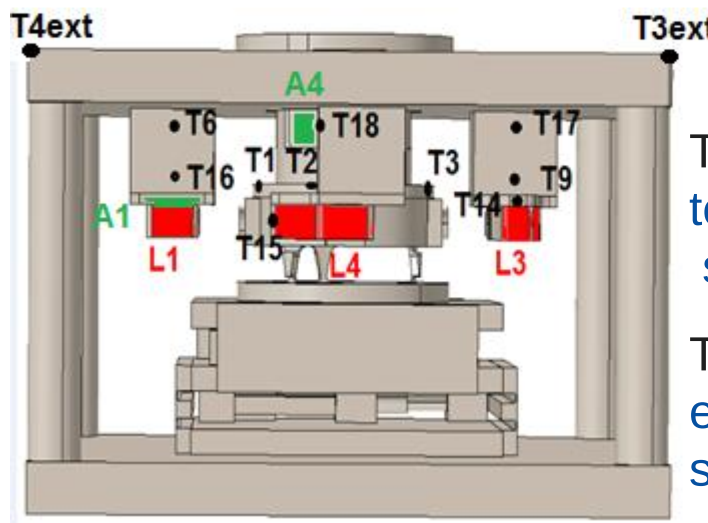


L1, L2, L3: laser interferometers

A1, .., A4: Actuators

P1, P2,P3: heating wires (disturbances)

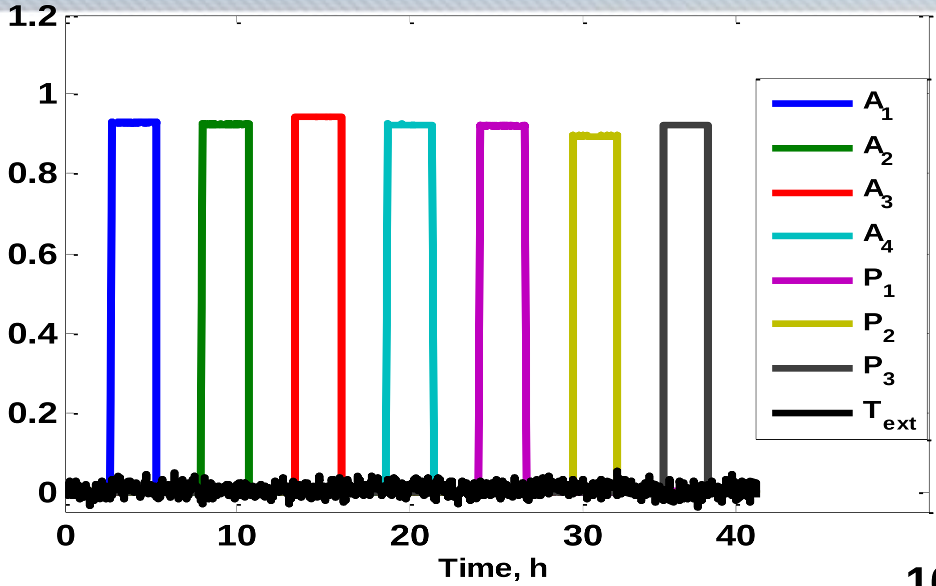




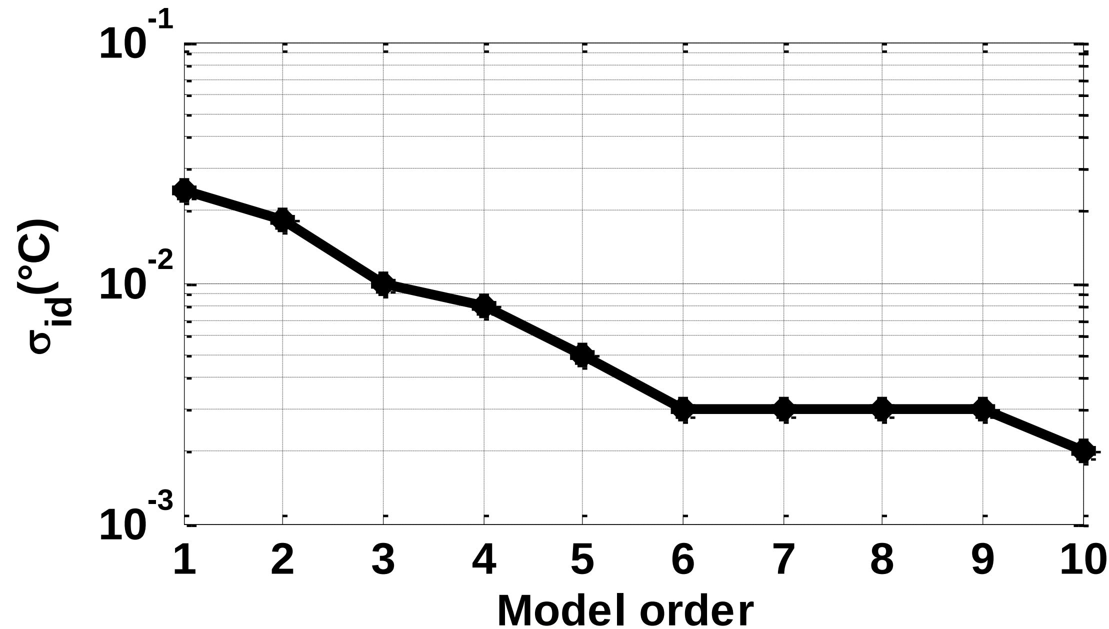
T1,..,T19: inner temperature sensors

T1ext, .., T4ext : external temperature sensors

Identification of the RM



$$U(t) = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ P_1 \\ P_2 \\ P_3 \\ T_{ext} \end{bmatrix} \qquad Y(t) = \begin{bmatrix} T_1 \\ \vdots \\ T_{19} \end{bmatrix}$$

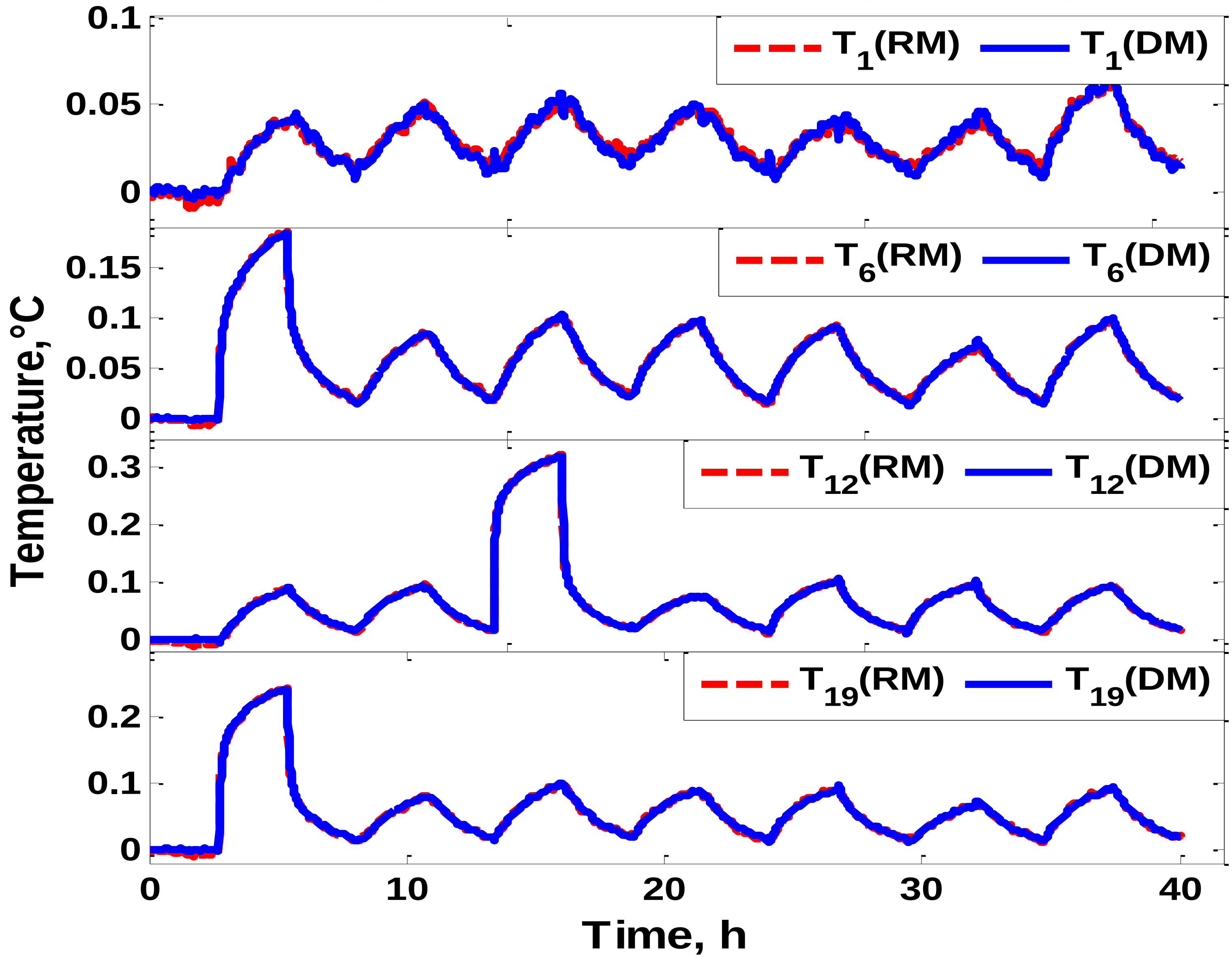


Optimization criterion:

$$\sigma_{id}{}^{n} = \sqrt{\frac{\sum_{i=1}^{q}\sum_{j=1}^{N_t}\left(Yrm_i(t_j) - Y_i^{DM}(t_j)\right)^2}{q \times N_t}}$$

Where:
q is the number of observables
N_t the number of time steps

Identification of the RM



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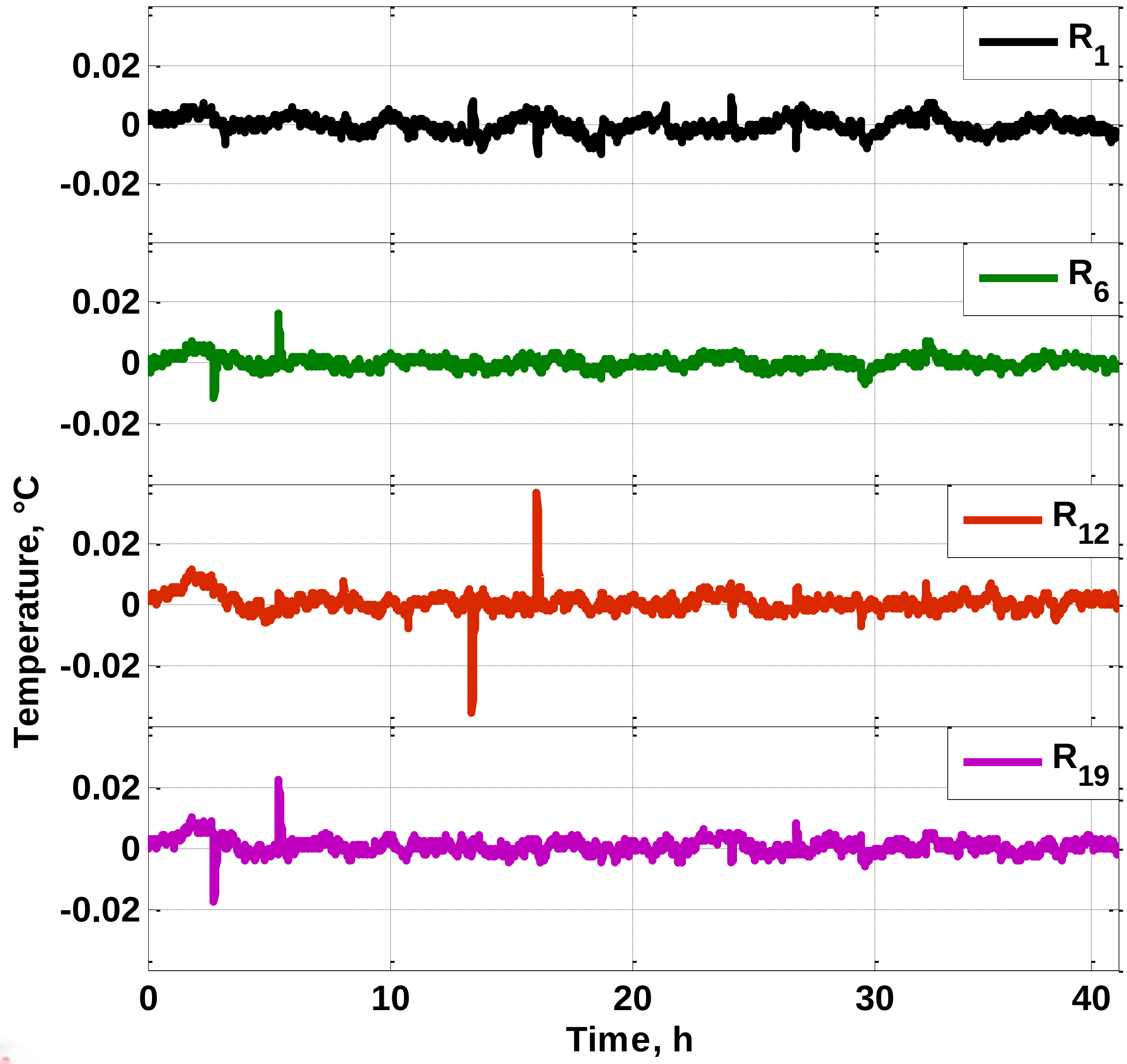
Evolution of the residuals (DM and RM)



Mean:
m =0.0005371 °C

Standard deviation:
σ =0.00253 °C

State feedback control:

$$\begin{cases} \dot{X}(t) = FX(t) + G_A U_A(t) + G_P U_P(t) \\ Y^{RM}(t) = HX(t) \end{cases}$$

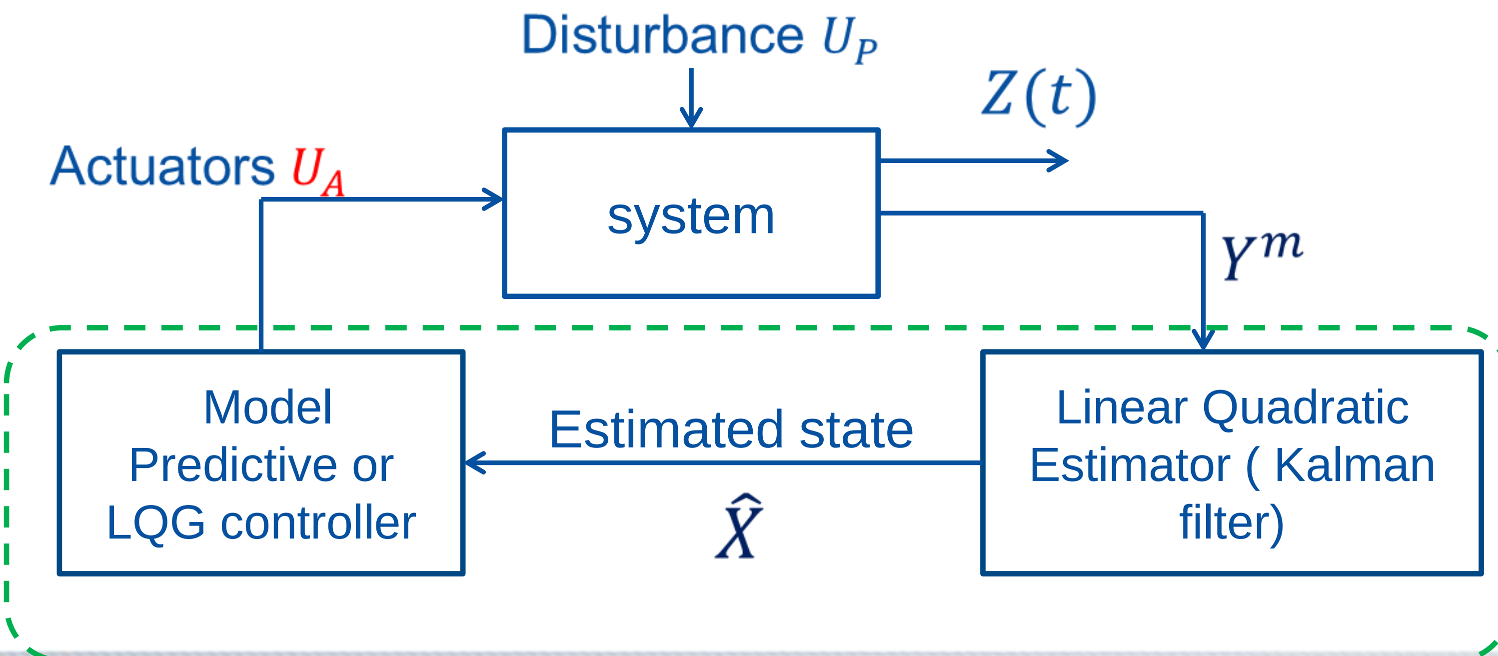
Temperatures to control :

$$\boldsymbol{Z(t) = H_z X(t)}$$

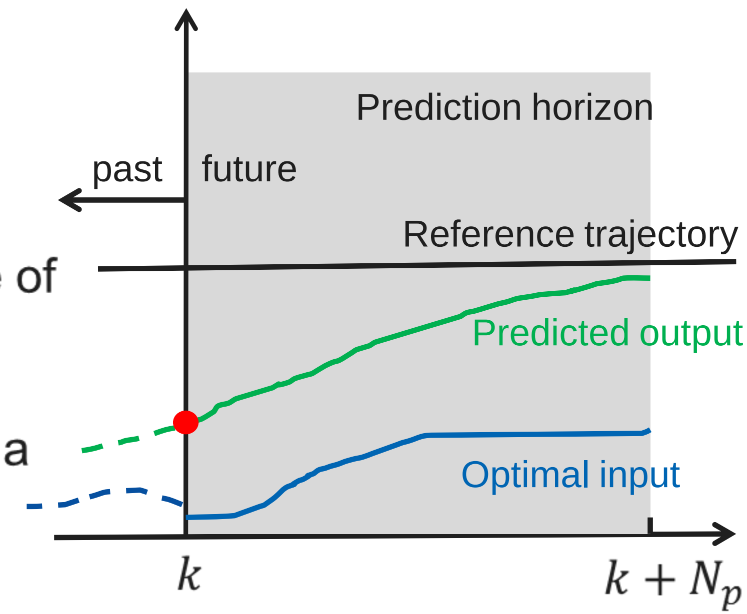
Where :

$\boldsymbol{U_A(t)}$: input vector of actuators

$\boldsymbol{U_P(t)}$: input vector of perturbations



Reduced Model



Principle :
A model of the system is used to predict plants behavior and choose the best control in the sense of some cost function within constraints.

The future response of the plant is predicted over a Prediction horizon N_p.

Dynamical system issued from time discretization of the state-space representation:

$$\boldsymbol{Z}(k) = \boldsymbol{\Psi} X(k) + \boldsymbol{\Gamma} U_A(k-1) + \boldsymbol{\Theta} \boldsymbol{U_A}(k)$$

- *k is the current time index*
- *N_p is the prediction horizon*

$$\boldsymbol{Z}(k) = \begin{bmatrix} Z(k+1) \\ \vdots \\ Z(k+N_p) \end{bmatrix} \qquad \boldsymbol{U_A}(k) = \begin{bmatrix} \Delta U_A(k) \\ \vdots \\ \Delta U_A(k+N_p-1) \end{bmatrix}$$

Model Predictive Control

The matrices Ψ, Γ, Θ depend on the matrices F,G,H of the reduced model.

The performance index to minimize :

$$J = \Delta\boldsymbol{Z}^T \Delta\boldsymbol{Z} + \lambda \boldsymbol{U_A}^T \boldsymbol{U_A}$$

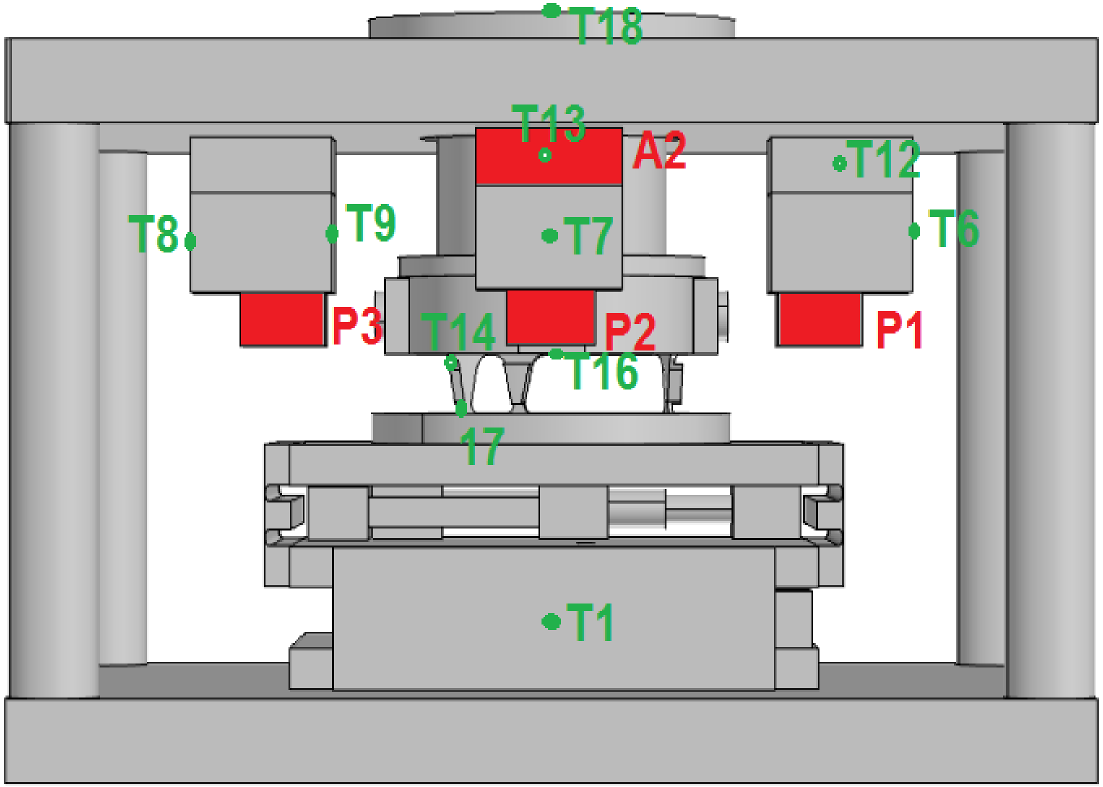
The control law issued from the minimization of a quadratic functional is :

$$\boldsymbol{U_A}(k) = (\boldsymbol{\Theta}^T\boldsymbol{\Theta} + \lambda I)^{-1}\boldsymbol{\Theta}^T\left[\boldsymbol{Z}_{ref}(k) - \boldsymbol{\Psi}X(k) - \boldsymbol{\Gamma}U_A(k-1)\right]$$

λ is a penalty parameter.

$X(k)$ is obtained by using a linear quadratic estimator (Kalman filter)

Control test case :



Control parameters :

Temperatures to control:

T_5, T_6, T_7, T_8

Reduced model:

RM10

Control time step:

$$\Delta t = 1s$$

Prediction horizon:

$$N_p = 1$$

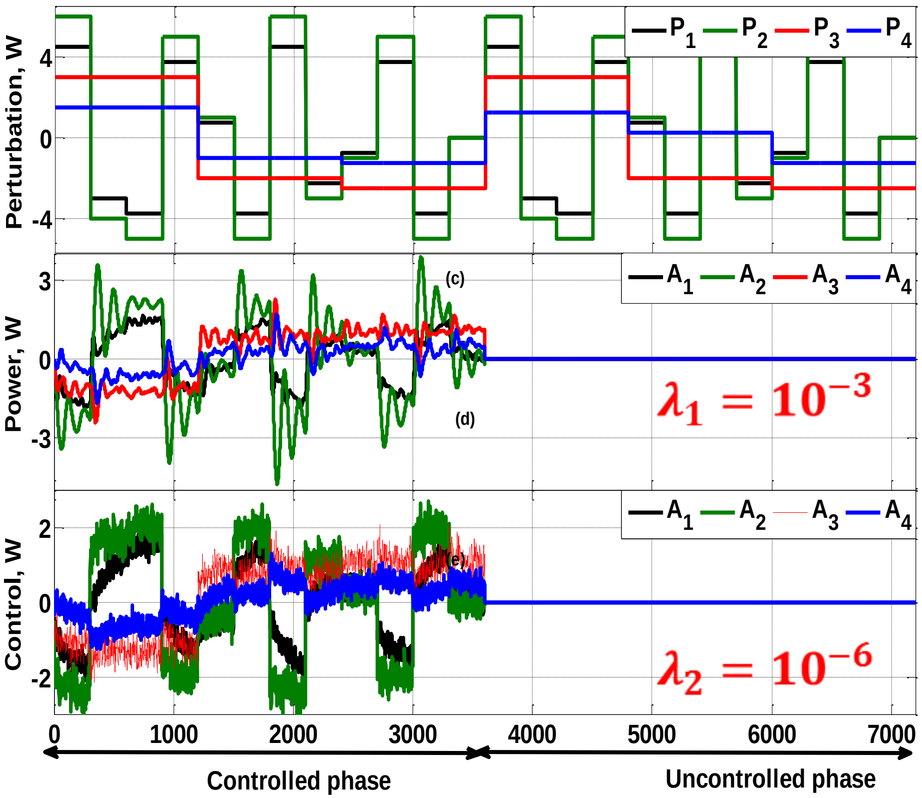
Standard deviation of the measurement noise:

$$\sigma_m = 0.002\ K$$

The mean quadratic discrepancy between the desired (0 K) and the obtained temperatures deviations:

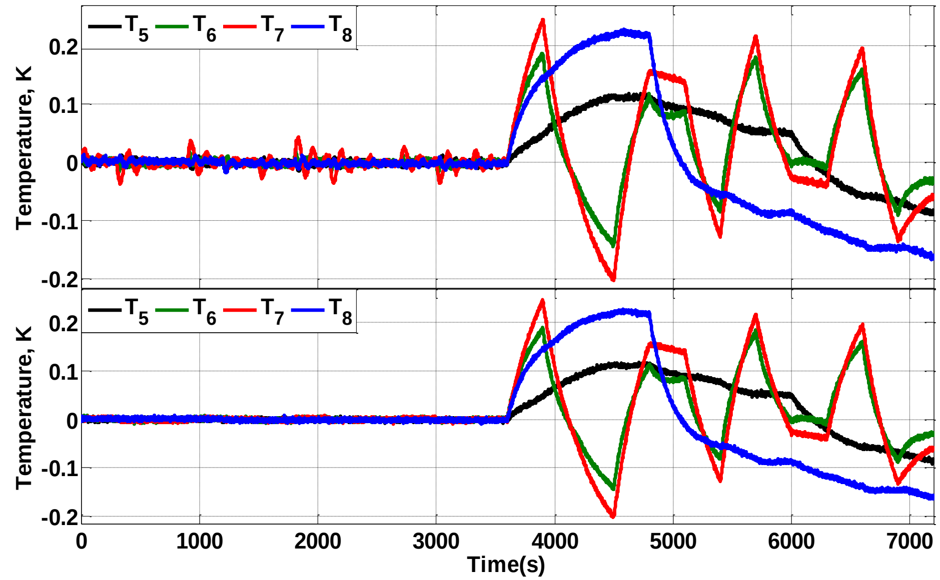
$$\sigma_z = \left(\frac{1}{4 \times N_t} \sum_{j=1}^{N_t} \sum_{i \in \{1,4\}} (Z_i(t_j))^2 \right)^{1/2}$$

Test case 1



Standard deviation of the perturbation:

$$\sigma_p = 2.99\ W$$



	Without control	With control	
σ_z, K	0.1051	0.0065	0.0024

Conclusions

- The thermal disturbances sources of the measurement machine were defined
- A reduced model built with the MIM from experimental data
- Thermal regulation of 4 temperature achieved by using an MPC controller

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Thank you for your attention

Question ?

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Linear quadratic estimator

State estimation:

$$\dot{\hat{X}}(t) = F\hat{X}(t) + G_A U_A(t) + K_f (Y^{RM}(t) - H\hat{X})$$

The correction is done through the Kalman gain given by :

$$K_f = \frac{1}{\alpha^2} S H^T$$

Where S ($\in \mathbb{R}^{n\times n}$) is the solution of Riccati equation:

$$SF^T + FS - \frac{1}{\alpha^2} SH^T HS + G_P {G_P}^T = 0$$

$\alpha = \frac{\sigma_m}{\sigma_p}$ is the ratio between the standard deviations of measurements and heat source disturbances

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