Title: Model reduction and thermal regulation by Model Predictive Control of a new cylindricity measurement apparatus

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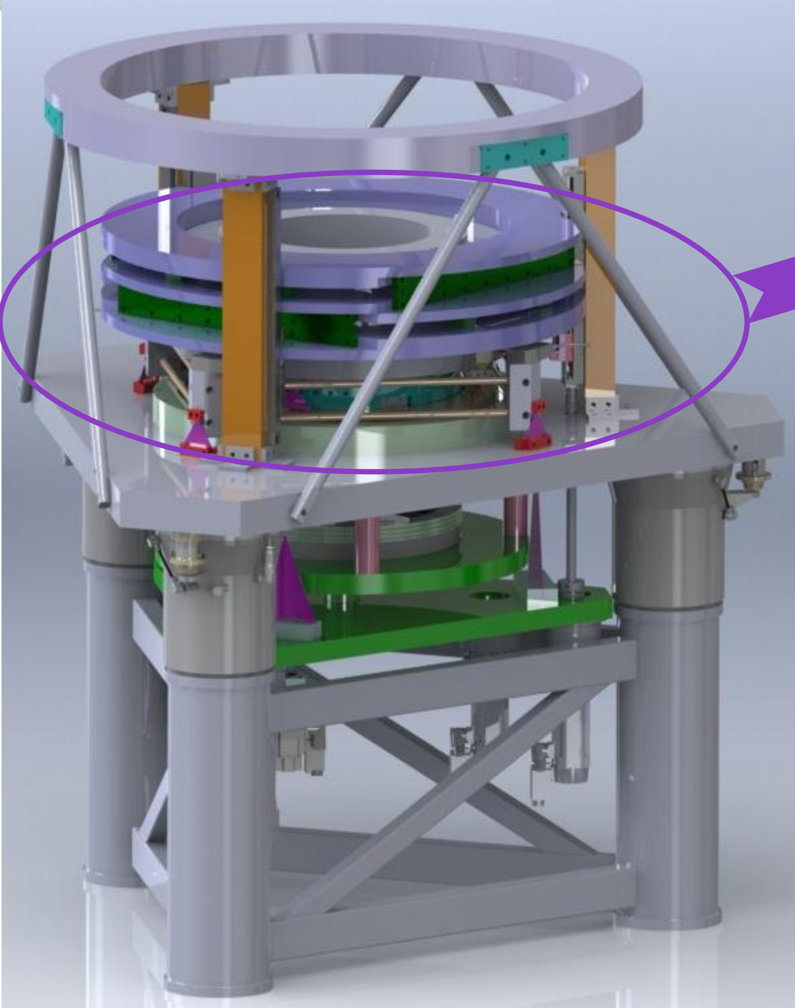
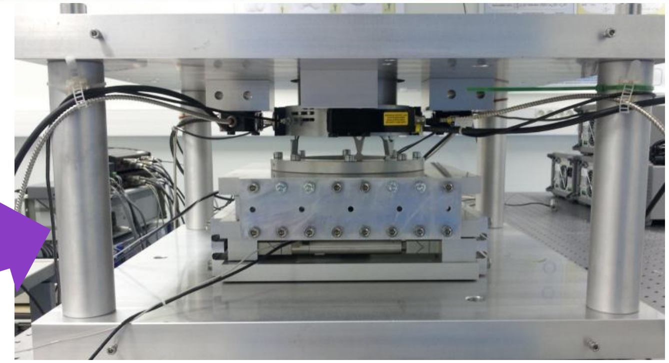
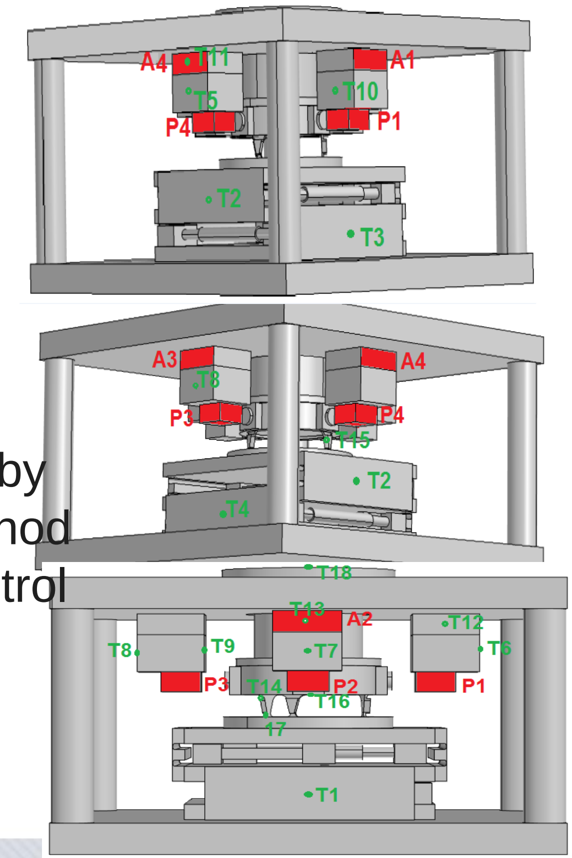
Context







Issues :
- Heat dissipation
- Thermal dilatation

Tools:
- Reduced model built by Modal Identification Method
- Model Predictive Control

Objective :
Real time control to reduce the effects of temperature variation

Model reduction

Thermal modeling :

Energy balance:

$$\vec{\nabla}.\left(k(M)\vec{\nabla}T(M,t)\right)+\sum_{j=1}^{n_Q}\left[\frac{P_j(t)}{V_j}\chi_j(M)\right]=\rho(M)C_p(M)\frac{\partial T}{\partial t}(M,t),$$

$$\forall\, M \in \Omega$$

Boundary conditions :

$$k(M)\vec{\nabla}T(M,t).\vec{n}=h(T_a(t)-T(M,t)),\forall M\in\Gamma$$

$$k(M)\vec{\nabla}T(M,t).\vec{n}=\frac{A_i(t)\xi_i(M)}{S_i}$$

Resolution of the heat transfer equation

Spatial discretization:
- Finite elements
- Finite volumes
- Finite differences

State space representation:

$$\begin{cases}\dot{\boldsymbol{T}}=\boldsymbol{AT}(\boldsymbol{t})+\boldsymbol{BU}(\boldsymbol{t})\\ \boldsymbol{Y}(\boldsymbol{t})=\boldsymbol{CT}(\boldsymbol{t})\end{cases}$$

N ODE

Issues:

- Memory
- Large computation time
- Unsuitable for real-time control

Solution

- Find a model reproducing the behaviour of the system with a small number of differential equations (model reduction)

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Model Identification Method (MIM)

1. Definition of a suitable structure of the reduced model

Modal state-space representation

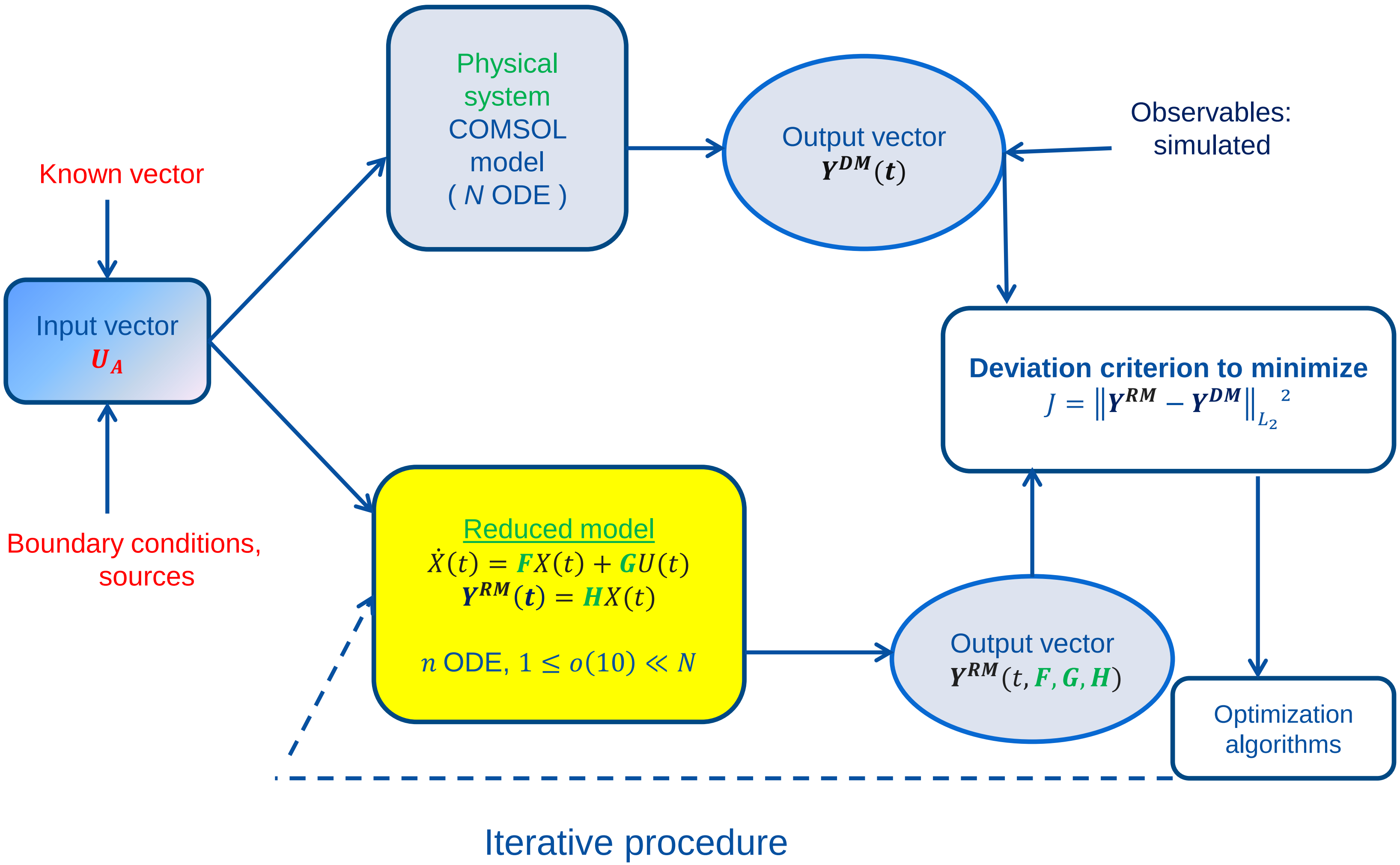
$$\begin{cases} \dot{X}(t) = \boldsymbol{F}X(t) + \boldsymbol{G}U(t) \\ \boldsymbol{Y}(\boldsymbol{t}) = \boldsymbol{H}X(t) \end{cases}$$ n ODE, n << N

2. Generation of numerical output data for a set of known input signals

3. Identification of the parameters of the reduced model (F,G,H) through optimization algorithms:

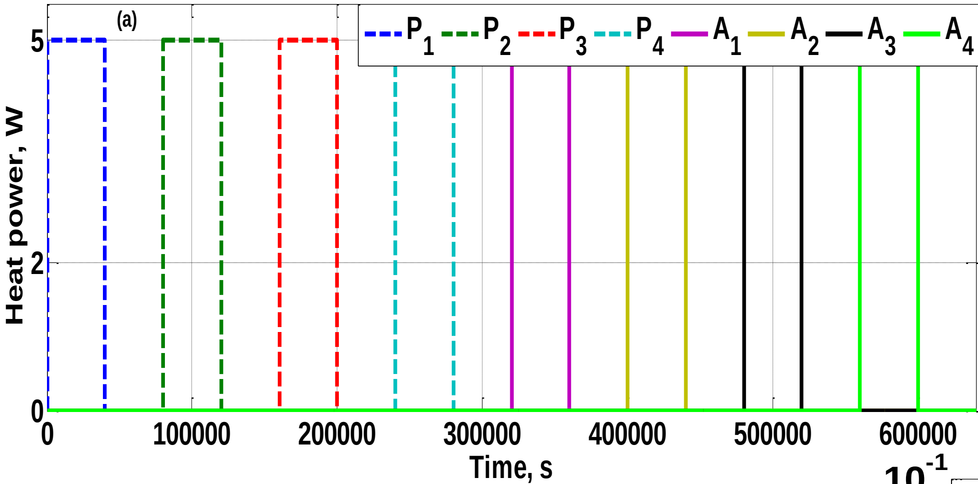
- Ordinary Linear Least Squares ($\boldsymbol{H}$)
- Particle Swarm Optimization ($\boldsymbol{F}$, $\boldsymbol{G}$)

Model Identification Method



Model Identification Method scheme

Identification of the RM



$$U(t) = \begin{bmatrix} A_1(t) \\ \vdots \\ A_4(t) \\ P_1(t) \\ \vdots \\ P_4(t) \end{bmatrix} \qquad \boldsymbol{Y}(t) = \begin{bmatrix} T_1 \\ \vdots \\ T_{18} \end{bmatrix}$$

Quadratic criterion:

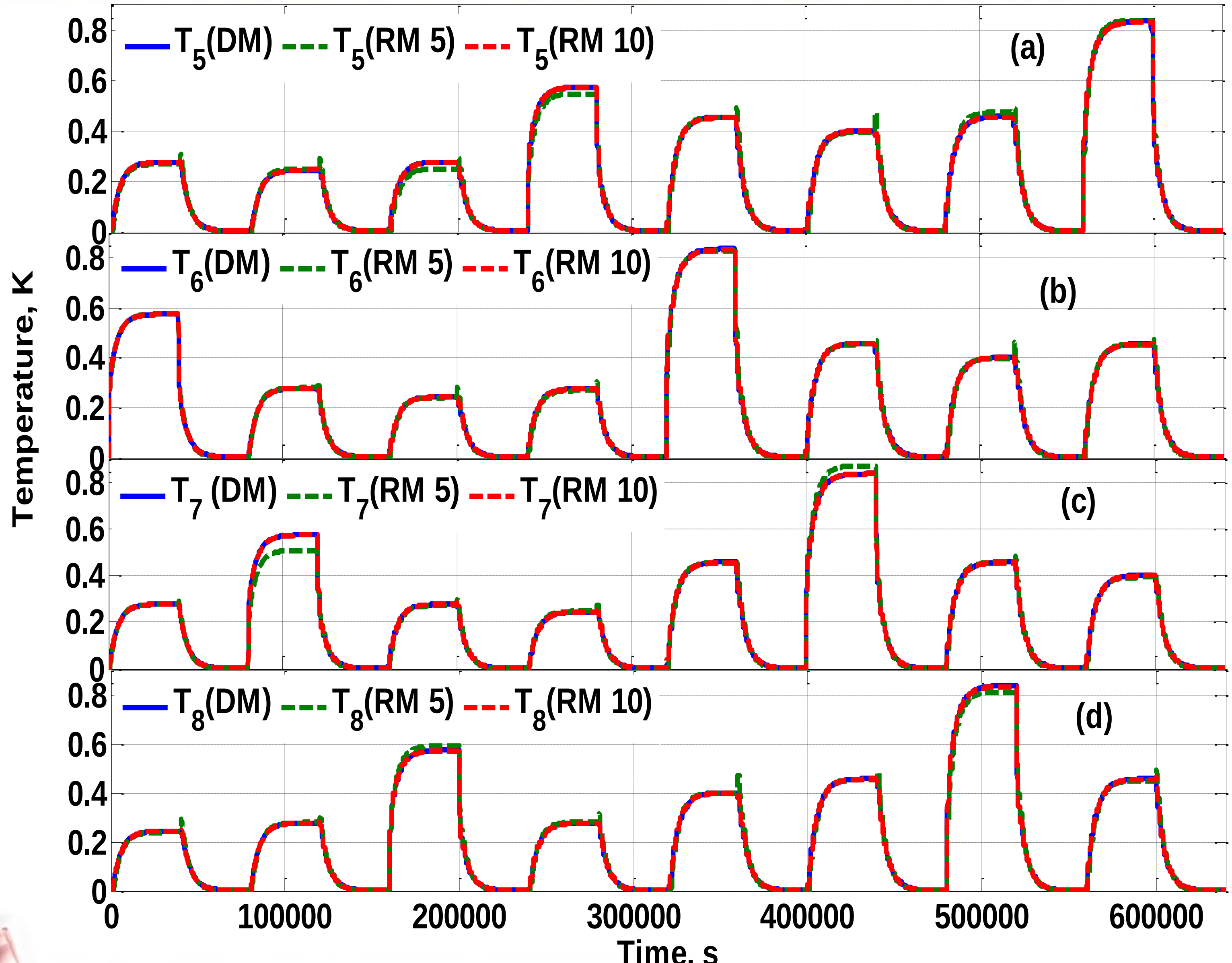
$$\boldsymbol{\sigma_{id}}^{\boldsymbol{n}} = \sqrt{\frac{\sum_{i=1}^{q}\sum_{j=1}^{N_t}(\boldsymbol{Yrm_i}(\boldsymbol{t_j}) - \boldsymbol{Y_i^{DM}}(\boldsymbol{t_j}))^2}{\boldsymbol{q} \times \boldsymbol{N_t}}}$$

Where:

q is the number of observables

N_t the number of time steps

Identification of the RM



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State feedback control:

$$\begin{cases} \dot{X}(t) = FX(t) + G_A U_A(t) + G_P U_P(t) \\ Y^{RM}(t) = HX(t) \end{cases}$$

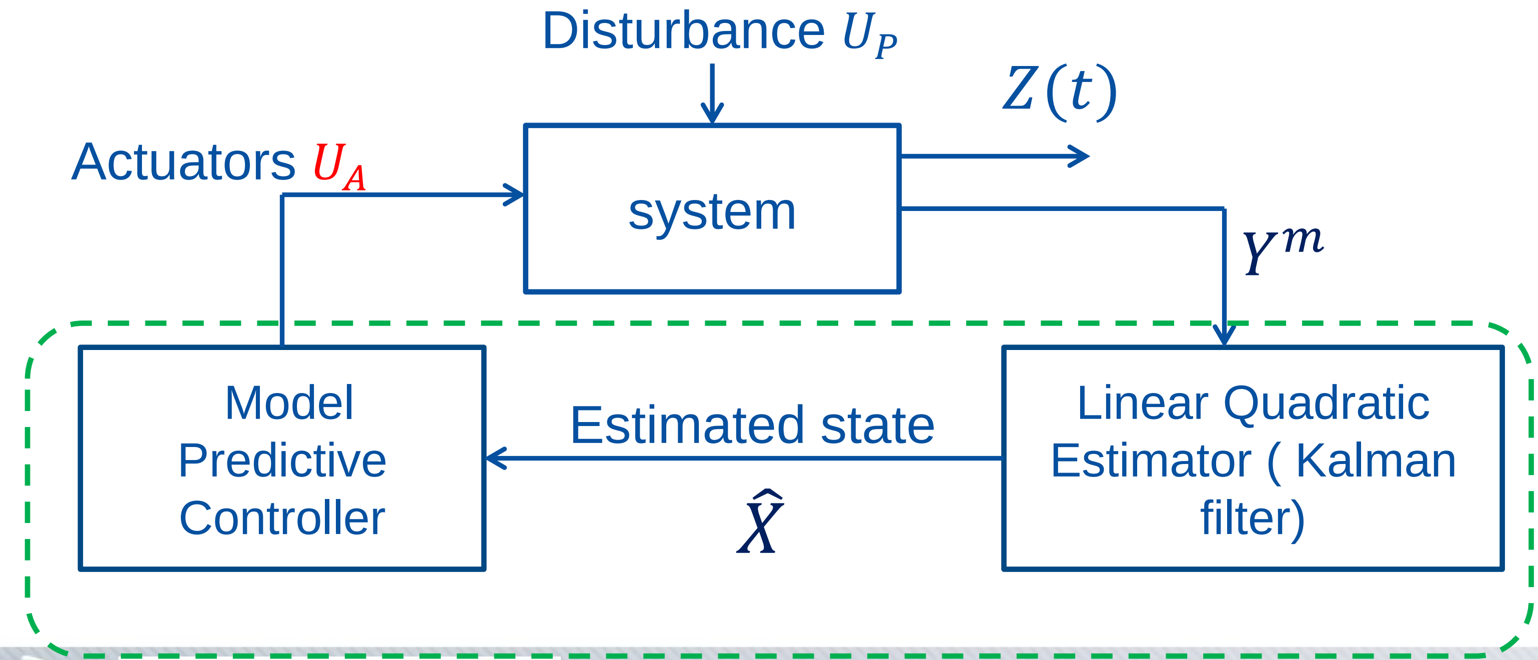
Temperatures to control :

$$Z(t) = [T_5, T_6, T_7, T_8]^T = H_Z X(t)$$

Where :

$U_A(t) = [A_1(t), A_2(t), A_3(t), A_4(t)]^T$ = input vector of actuators (surface heaters)

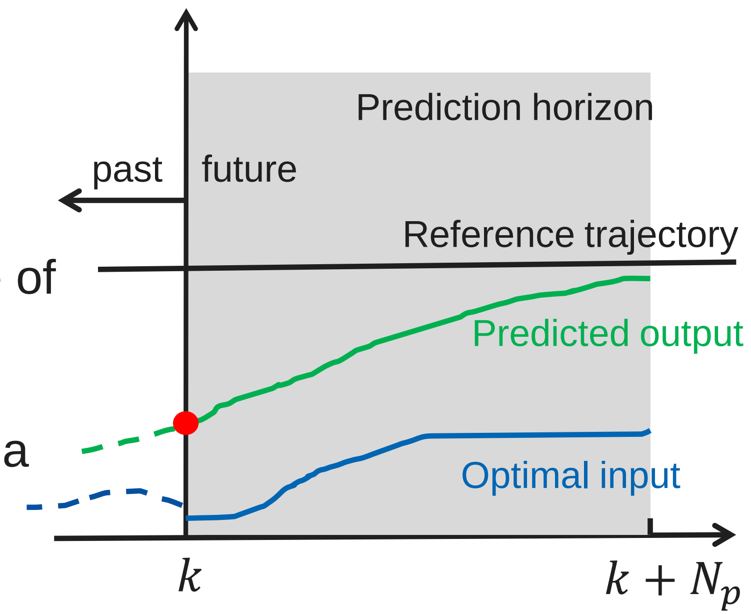
$U_P(t) = [P_1(t), P_2(t), P_3(t), P_4(t)]^T$ = input vector of perturbations (laser interferometers)



Reduced Model

<u>Principle :</u>
A model of the system is used to predict plants behavior and choose the best control in the sense of some cost function within constraints.

The future response of the plant is predicted over a Prediction horizon N_p.



Dynamical system issued from time discretization of the state-space representation:

$$\boldsymbol{Z}(k) = \boldsymbol{\Psi} X(k) + \boldsymbol{\Gamma} U_A(k-1) + \boldsymbol{\Theta} \boldsymbol{U_A}(k)$$

- *k is the current time index*
- *N_p is the prediction horizon*

$$\boldsymbol{Z}(k) = \begin{bmatrix} Z(k+1) \\ \vdots \\ Z(k+N_p) \end{bmatrix} \qquad \boldsymbol{U_A}(k) = \begin{bmatrix} \Delta U_A(k) \\ \vdots \\ \Delta U_A(k+N_p-1) \end{bmatrix}$$

The matrices Ψ, Γ, Θ depend on the matrices F,G,H of the reduced model.

The performance index to minimize :

$$J = \Delta\boldsymbol{Z}^{T}\Delta\boldsymbol{Z} + \lambda\boldsymbol{U_A}^{T}\boldsymbol{U_A}$$

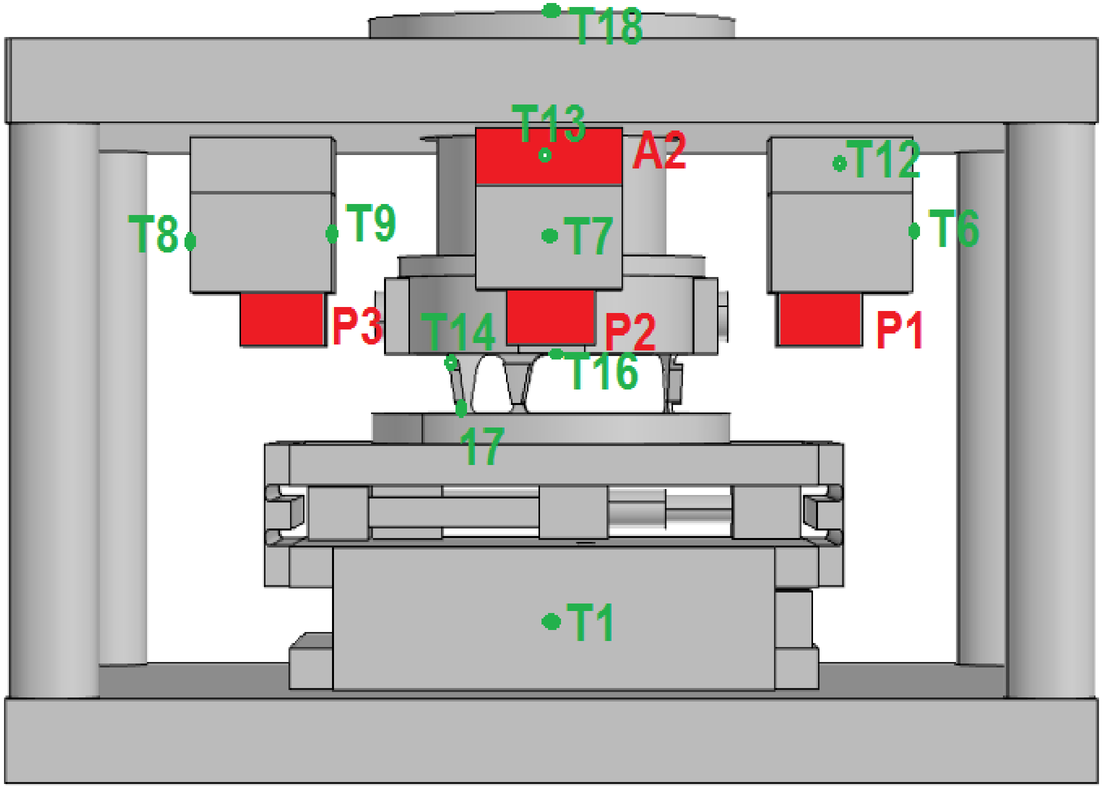
The control law issued from the minimization of a quadratic functional is :

$$\boldsymbol{U_A}(k) = (\boldsymbol{\Theta}^T\boldsymbol{\Theta} + \lambda I)^{-1}\boldsymbol{\Theta}^T\left[\boldsymbol{Z}_{ref}(k) - \boldsymbol{\Psi}X(k) - \boldsymbol{\Gamma}U_A(k-1)\right]$$

λ is a penalty parameter.

$X(k)$ is obtained by using a linear quadratic estimator (Kalman filter)

Control test case :



Control parameters :

Temperatures to control:

T_5, T_6, T_7, T_8

Reduced model:

RM10

Control time step:

$$\Delta t = 1s$$

Prediction horizon:

$$N_p = 1$$

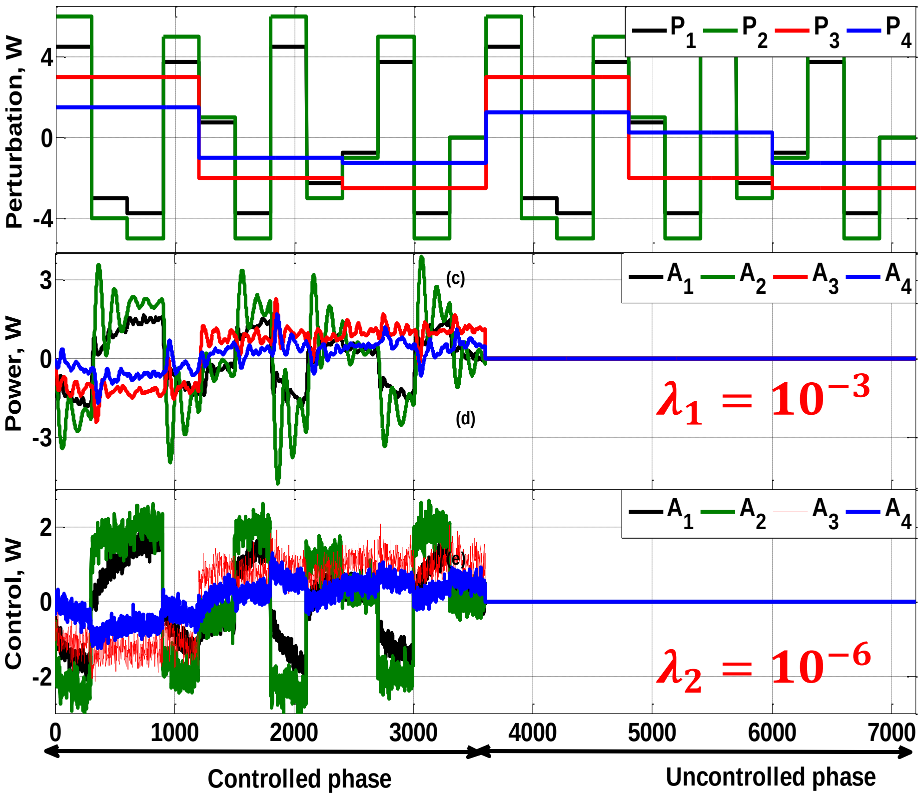
Standard deviation of the measurement noise:

$$\sigma_m = 0.002\ K$$

The mean quadratic discrepancy between the desired (0K) and the obtained temperatures deviations:

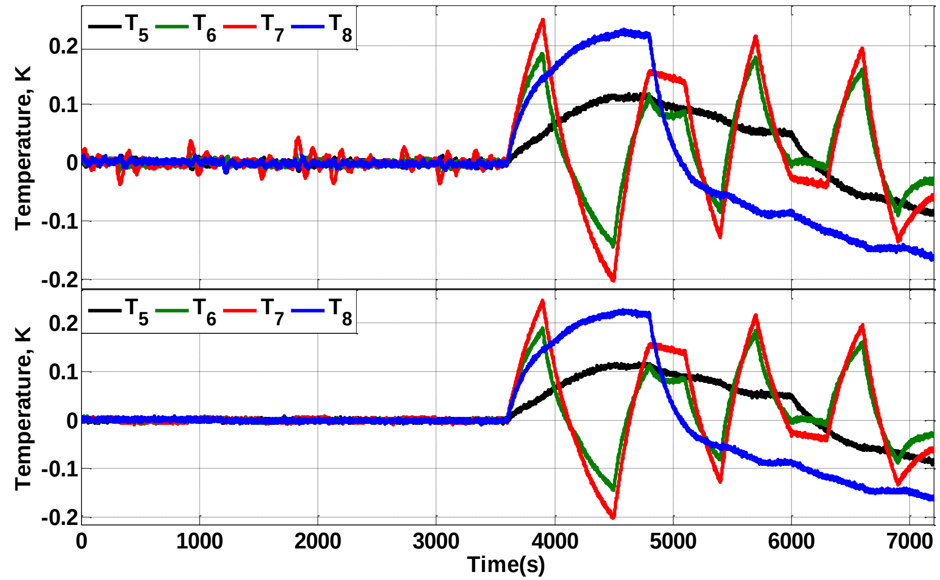
$$\sigma_z = \left(\frac{1}{4 \times N_t} \sum_{j=1}^{N_t} \sum_{i \in \{1,4\}} \left(Z_i(t_j)\right)^2 \right)^{1/2}$$

Test case 1



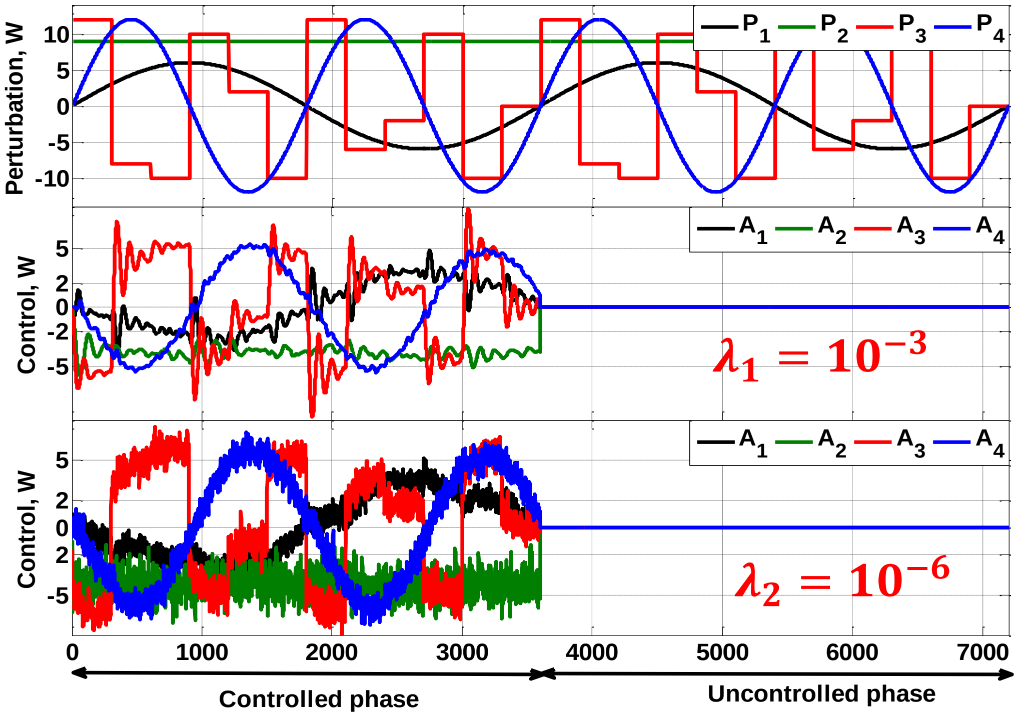
Standard deviation of the perturbation:

$$\sigma_p = 2.99\ W$$



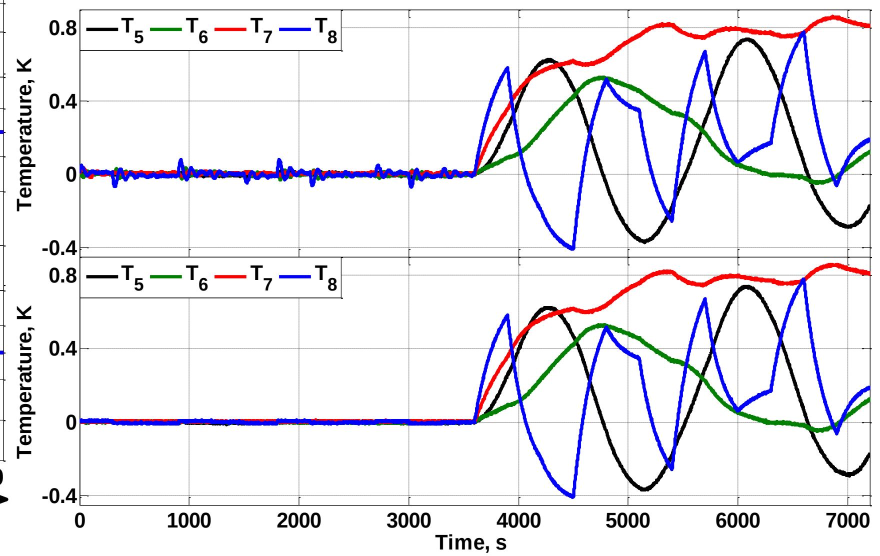
	Without control	With control	
σ_z, K	0.1051	0.0065	0.0024

Test case 2



Standard deviation of the perturbation:

$$\sigma_p = 6.42W$$



	Without control	With control	
σ_z, K	0.4572	0.0120	0.0041

Conclusions and prospects

- Numerical generation of input-output data
- A reduced model built with the MIM from numerical generation
- Thermal regulation of temperature in 4 points of the structure

- Study the effects of the MPC parameters
- Identify a reduced model from experimental data
- Increase the number of controlled points

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Linear quadratic estimator

State estimation:

$$\dot{\hat{X}}(t) = F\hat{X}(t) + G_A U_A(t) + K_f(Y^{RM}(t) - H\hat{X})$$

The correction is done through the Kalman gain given by :

$$K_f = \frac{1}{\alpha^2} S H^T$$

Where S ($\in \mathbb{R}^{n\times n}$) is the solution of Riccati equation:

$$SF^T + FS - \frac{1}{\alpha^2} SH^T HS + G_P {G_P}^T = 0$$

$\alpha = \frac{\sigma_m}{\sigma_p}$ is the ratio between the standard deviations of measurements and heat source disturbances