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Investigation of Calibration Methods for Multiport VNA Measurements

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Outline

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   - Conventional vs. generalised S-parameters
2. Full Multiport Calibration Method
   - Error term model description
   - Calibration procedure
   - Measurement results
3. Port-to-Port Multiport Calibration Method
   - Procedure description
   - Measurement results

S-Parameter Measurements (N-port case)

- Conventional (ideal) S-Parameters consider solely incident signals a_i of source ports for each switch position (*I, II, ..., N*)

$$[S]=\begin{bmatrix} b_1^I & b_1^{II} & \cdots & b_1^N \\ b_2^I & b_2^{II} & \cdots & b_2^N \\ \vdots & \vdots & \ddots & \\ b_N^I & b_N^{II} & \cdots & b_N^N \end{bmatrix} \cdot \begin{bmatrix} a_1^I & 0 & \cdots & 0 \\ 0 & a_2^{II} & \cdots & 0 \\ \vdots & \vdots & \ddots & \\ 0 & 0 & \cdots & a_N^N \end{bmatrix}^{-1}$$

- Generalised S-Parameters (denoted by *M*) consider incident signals m_{ai} of all ports for each switch position (*I, II, ..., N*) and removes port match errors (Switch terms)

$$[M]=\begin{bmatrix} m_{b1}^I & m_{b1}^{II} & \cdots & m_1^N \\ m_{b2}^I & m_{b2}^{II} & \cdots & m_{b2}^N \\ \vdots & \vdots & \ddots & \\ m_{bN}^I & m_{bN}^{II} & \cdots & m_{bN}^N \end{bmatrix} \cdot \begin{bmatrix} m_{a1}^I & m_{a1}^{II} & \cdots & m_{a1}^N \\ m_{a2}^I & m_{a2}^{II} & \cdots & m_{a1}^N \\ \vdots & \vdots & \ddots & \vdots \\ m_{aN}^I & m_{aN}^{II} & \cdots & m_{aN}^N \end{bmatrix}^{-1}$$

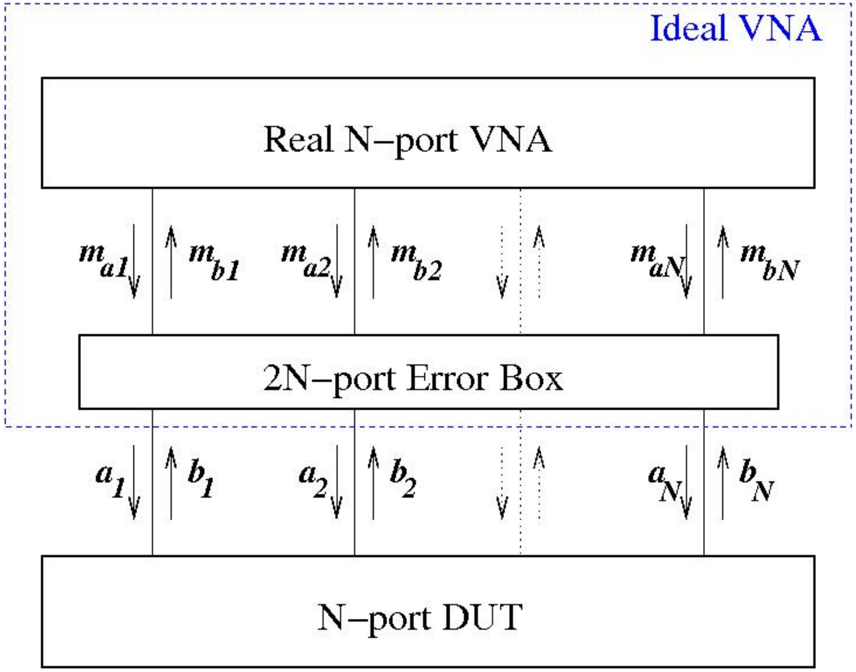


Fig.1: Generalised model of a multiport test set

S-Parameter Measurements (2-port case)

- Conventional (ideal) S-Parameters:

$$[S]=\begin{bmatrix} b_1^I & b_1^{II} \\ b_2^I & b_2^{II} \end{bmatrix}\cdot\begin{bmatrix} a_1^I & 0 \\ 0 & a_2^{II} \end{bmatrix}^{-1}=\begin{bmatrix} b_1^I & b_1^{II} \\ b_2^I & b_2^{II} \end{bmatrix}\cdot\begin{bmatrix} \dfrac{1}{a_1^I} & 0 \\ 0 & \dfrac{1}{a_2^{II}} \end{bmatrix}=\begin{bmatrix} \dfrac{b_1^I}{a_1^I} & \dfrac{b_1^{II}}{a_2^{II}} \\ \dfrac{b_2^I}{a_1^I} & \dfrac{b_2^{II}}{a_2^{II}} \end{bmatrix}$$

- Generalised S-Parameters:

$$[M]=\begin{bmatrix} m_{b1}^I & m_{b1}^{II} \\ m_{b2}^I & m_{b2}^{II} \end{bmatrix}\cdot\begin{bmatrix} m_{a1}^I & m_{a1}^{II} \\ m_{a2}^I & m_{a2}^{II} \end{bmatrix}^{-1}=\begin{bmatrix} \dfrac{\dfrac{m_{b1}^I}{m_{a1}^I}-\dfrac{m_{b2}^I}{m_{a1}^I}\dfrac{m_{b1}^{II}}{m_{a2}^{II}}\Gamma_2}{d} & \dfrac{\dfrac{m_{b1}^{II}}{m_{a2}^{II}}-\dfrac{m_{b1}^I}{m_{a1}^I}\dfrac{m_{b1}^{II}}{m_{a2}^{II}}\Gamma_1}{d} \\ \dfrac{\dfrac{m_{b2}^I}{m_{a1}^I}-\dfrac{m_{b2}^I}{m_{a1}^I}\dfrac{m_{b2}^{II}}{m_{a2}^{II}}\Gamma_2}{d} & \dfrac{\dfrac{m_{b2}^{II}}{m_{a2}^{II}}-\dfrac{m_{b2}^I}{m_{a1}^I}\dfrac{m_{b1}^{II}}{m_{a2}^{II}}\Gamma_1}{d} \end{bmatrix}$$

with abbreviations:

$$d=1-\frac{m_{b2}^I}{m_{a1}^I}\frac{m_{b1}^{II}}{m_{a2}^{II}}\Gamma_1\Gamma_2$$ and switch terms: $$\Gamma_1=\frac{m_{a1}^{II}}{m_{b1}^{II}} \quad , \quad \Gamma_2=\frac{m_{a2}^I}{m_{b2}^I}$$

- In the ideal case, where $\Gamma_1=\Gamma_2=0$, [*M*] reduces to [*S*].

Full Multiport Least Squares Calibration Method

- Applies a $(4N^2-1)$-term error model.
- Requires dual reflectometers (couplers) at each test port to measure the generalised S-parameters (*M*) .
- For the sake of clarity the formalism is described for 2-port case:

  - The corrected signals *a* and *b* are related to the raw signals *m* by the error term matrix [*C*] as follows:

$$\begin{pmatrix} b_1 \\ b_2 \\ a_1 \\ a_2 \end{pmatrix} = [C] \begin{pmatrix} m_{a1} \\ m_{a2} \\ m_{b1} \\ m_{b2} \end{pmatrix} = \begin{bmatrix} [G] & [E] \\ [F] & [H] \end{bmatrix} \begin{pmatrix} m_{a1} \\ m_{a2} \\ m_{b1} \\ m_{b2} \end{pmatrix} \Rightarrow \begin{cases} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = [G] \begin{pmatrix} m_{a1} \\ m_{a2} \end{pmatrix} + [E] \begin{pmatrix} m_{b1} \\ m_{b2} \end{pmatrix} & (1) \\ \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = [F] \begin{pmatrix} m_{a1} \\ m_{a2} \end{pmatrix} + [H] \begin{pmatrix} m_{b1} \\ m_{b2} \end{pmatrix} & (2) \end{cases}$$

  - By substituting Eqs. (1) and (2) into the expression for S-parameters, we get the fundamental equation for each switch position!

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = [S] \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \Rightarrow [G] \begin{pmatrix} m_{a1} \\ m_{a2} \end{pmatrix} + [E] \begin{pmatrix} m_{b1} \\ m_{b2} \end{pmatrix} = [S] \left\{ [F] \begin{pmatrix} m_{a1} \\ m_{a2} \end{pmatrix} + [H] \begin{pmatrix} m_{b1} \\ m_{b2} \end{pmatrix} \right\}$$

Full Multiport Least Squares Calibration Method

- Combining the equations for the two switch positions *I* and *II*:

$$[G]\begin{bmatrix} m_{a1}^{I} & m_{a1}^{II} \\ m_{a2}^{I} & m_{a2}^{II} \end{bmatrix} + [E]\begin{bmatrix} m_{b1}^{I} & m_{b1}^{II} \\ m_{b2}^{I} & m_{b2}^{II} \end{bmatrix} = [S]\left\{[F]\begin{bmatrix} m_{a1}^{I} & m_{a1}^{II} \\ m_{a2}^{I} & m_{a2}^{II} \end{bmatrix} + [H]\begin{bmatrix} m_{b1}^{I} & m_{b1}^{II} \\ m_{b2}^{I} & m_{b2}^{II} \end{bmatrix}\right\}$$

and substituting the expression for the generalised S-parameters:

$$[M] = \begin{bmatrix} m_{b1}^{I} & m_{b1}^{II} \\ m_{b2}^{I} & m_{b2}^{II} \end{bmatrix}\begin{bmatrix} m_{a1}^{I} & m_{a1}^{II} \\ m_{a2}^{I} & m_{a2}^{II} \end{bmatrix}^{-1} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$$

leads to the final equation:

$$[G] + [E][M] = [S]\{[F] + [H][M]\} \qquad (3)$$

- Equation (3) is applied for
  - calculating error terms from calibration measurements and subsequently
  - calculating S-parameters from DUT measurements.
- Equation (3) can be rearranged to form a linear equation system, solved for the error terms [*G*], [*E*], [*F*] and [*H*].

Full Multiport Least Squares Calibration Method

- Each calibration measurement (*i*) yields:

$$-\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} - \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix}\begin{bmatrix} m_{11}^{i} & m_{12}^{i} \\ m_{21}^{i} & m_{22}^{i} \end{bmatrix} + \begin{bmatrix} S_{11}^{i} & S_{12}^{i} \\ S_{21}^{i} & S_{22}^{i} \end{bmatrix}\left\{\begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} + \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}\begin{bmatrix} m_{11}^{i} & m_{12}^{i} \\ m_{21}^{i} & m_{22}^{i} \end{bmatrix}\right\} = 0$$

- Evaluating matrix multiplications and sorting the coefficients of error terms in a coefficient matrix leads to:

$$\begin{bmatrix} -1 & 0 & 0 & 0 & -m_{11}^{i} & -m_{21}^{i} & 0 & 0 & S_{11}^{i} & 0 & S_{12}^{i} & 0 & S_{11}^{i}m_{11}^{i} & S_{11}^{i}m_{21}^{i} & S_{12}^{i}m_{11}^{i} & S_{12}^{i}m_{21}^{i} \\ 0 & -1 & 0 & 0 & -m_{12}^{i} & -m_{22}^{i} & 0 & 0 & 0 & S_{11}^{i} & 0 & S_{12}^{i} & S_{11}^{i}m_{12}^{i} & S_{11}^{i}m_{22}^{i} & S_{12}^{i}m_{12}^{i} & S_{12}^{i}m_{22}^{i} \\ 0 & 0 & -1 & 0 & 0 & 0 & -m_{11}^{i} & -m_{21}^{i} & S_{21}^{i} & 0 & S_{22}^{i} & 0 & S_{21}^{i}m_{11}^{i} & S_{21}^{i}m_{21}^{i} & S_{22}^{i}m_{11}^{i} & S_{22}^{i}m_{21}^{i} \\ 0 & 0 & 0 & -1 & 0 & 0 & -m_{12}^{i} & -m_{22}^{i} & 0 & S_{21}^{i} & 0 & S_{22}^{i} & S_{21}^{i}m_{12}^{i} & S_{21}^{i}m_{22}^{i} & S_{22}^{i}m_{12}^{i} & S_{22}^{i}m_{22}^{i} \end{bmatrix}\vec{e} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (4)$$

with error term vector:

$$\vec{e} = \left(G_{11},\, G_{12},\, G_{21},\, G_{22},\, E_{11},\, E_{12},\, E_{21},\, E_{22},\, F_{11},\, F_{12},\, F_{21},\, F_{22},\, H_{11},\, H_{12},\, H_{21},\, H_{22}\right)^{T}$$

- The homogeneous equation system (4) leads to the trivial zero solution!

Full Multiport Least Squares Calibration Method

- Normalising the error terms by setting G_{11} to 1 (e.g.) leads to the solvable inhomogeneous equation system:

$$\underbrace{\begin{bmatrix} 0 & 0 & 0 & -m_{11}^i & -m_{21}^i & 0 & 0 & S_{11}^i & 0 & S_{12}^i & 0 & S_{11}^i m_{11}^i & S_{11}^i m_{21}^i & S_{12}^i m_{11}^i & S_{12}^i m_{21}^i \\ -1 & 0 & 0 & -m_{12}^i & -m_{22}^i & 0 & 0 & 0 & S_{11}^i & 0 & S_{12}^i & S_{11}^i m_{12}^i & S_{11}^i m_{22}^i & S_{12}^i m_{12}^i & S_{12}^i m_{22}^i \\ 0 & -1 & 0 & 0 & 0 & -m_{11}^i & -m_{21}^i & S_{21}^i & 0 & S_{22}^i & 0 & S_{21}^i m_{11}^i & S_{21}^i m_{21}^i & S_{22}^i m_{11}^i & S_{22}^i m_{21}^i \\ 0 & 0 & -1 & 0 & 0 & -m_{12}^i & -m_{22}^i & 0 & S_{21}^i & 0 & S_{22}^i & S_{21}^i m_{12}^i & S_{21}^i m_{22}^i & S_{22}^i m_{12}^i & S_{22}^i m_{22}^i \end{bmatrix}}_{A^i} \vec{e}\,' = \underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{B^i}$$

with modified error term vector:

$$\vec{e}\,' = \left(G_{12},\ G_{21},\ G_{22},\ E_{11},\ E_{12},\ E_{21},\ E_{22},\ F_{11},\ F_{12},\ F_{21},\ F_{22},\ H_{11},\ H_{12},\ H_{21},\ H_{22}\right)^T$$

- Combining the results of n>=5 calibration measurements leads to the least squares solution for the error terms.

$$\begin{bmatrix} [A^1] \\ \vdots \\ [A^i] \\ \vdots \\ [A^n] \end{bmatrix} \vec{e}\,' = \begin{pmatrix} (B^1) \\ \vdots \\ (B^i) \\ \vdots \\ (B^n) \end{pmatrix}$$

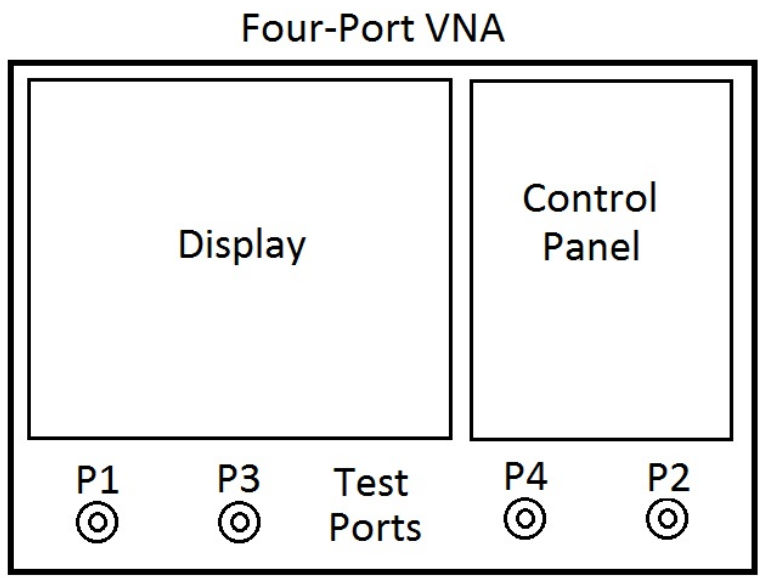
- Once the error terms have been determined, S-parameters of the DUT can be calculated applying:

$$[S_{DUT}] = \{[G] + [E][M_{DUT}]\}\{[F] + [H][M_{DUT}]\}^{-1}$$

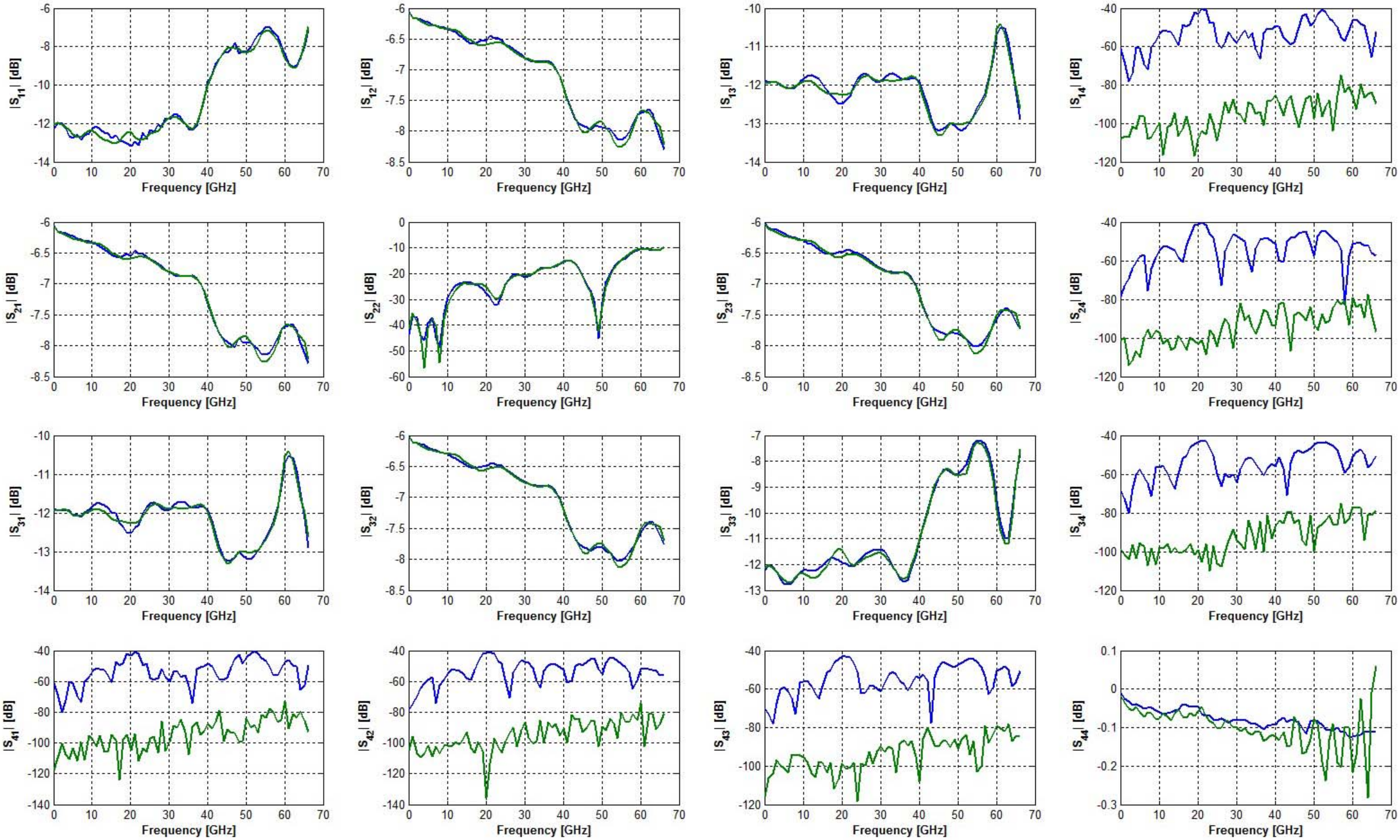
Four-port Calibration Procedure

- Due to the lack of real 4-port calibration standards, differing standards can be constructed by combining 1-port standards and thru connections:
  - Standards: Open (O), Offset shorts (S1 ... S4), Broadband Load (L)
  - DUT: Power splitter

Ports	P1 (m)	P3 (m)	P4 (f)	P2 (f)
Calibration Measurements	O	S1	S2	S3
	S1	S2	S3	S4
	S2	S3	S4	L
	S3	S4	L	O
	S4	L	O	S1
	L	O	S1	S
	P2	P4	P3	P1
	P4	P2	P1	P3
DUT	$PS_{out\ L}$	$PS_{out\ R}$	S1	PS_{in}

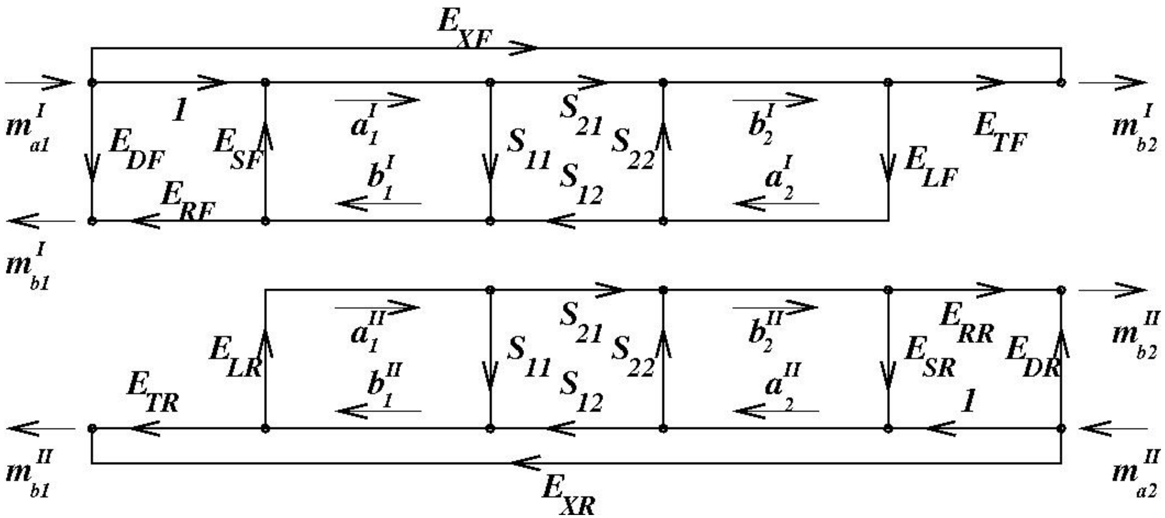


Measurement results. Comparison: Full 4-port LSQ vs. 2-port ECal



Port-to-Port Multiport Calibration Procedure

- Applies common 2-port 12-term error model:

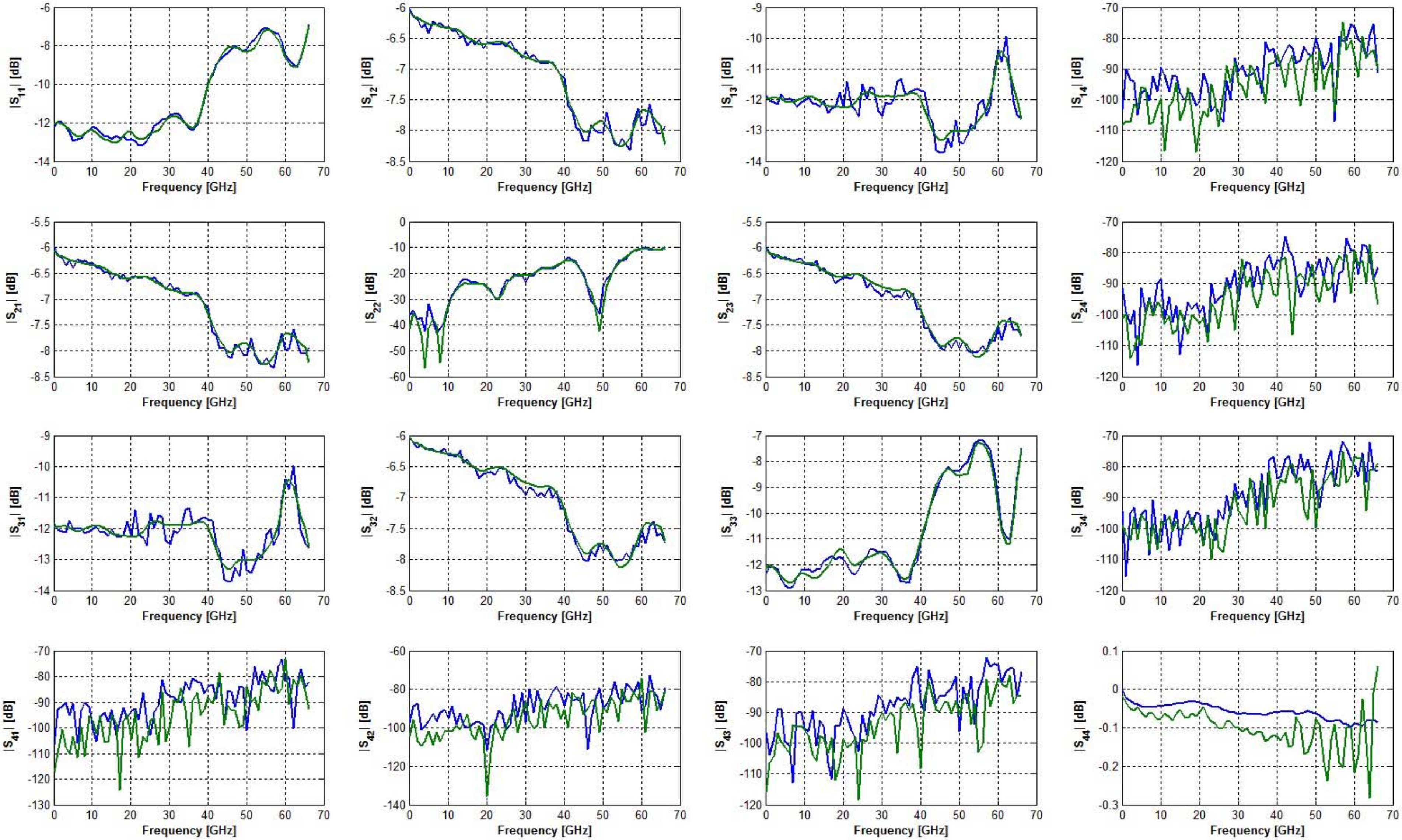


Forward direction

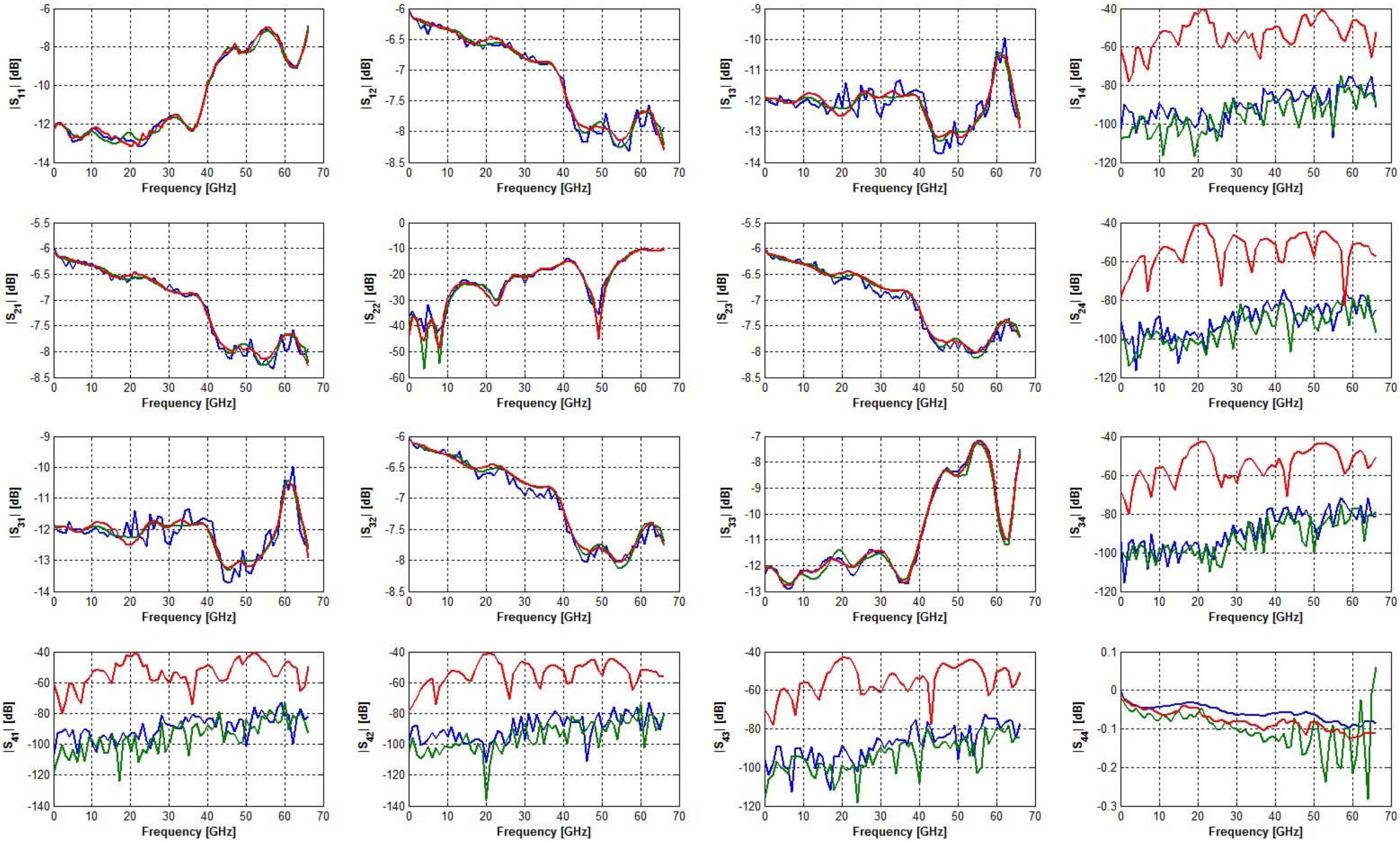
Reverse direction

- Relies on successive determination of error terms with the help of appropriate calibration measurements:
    - N 1-port calibrations to determine directivity, source match and reflection tracking terms for each port.
    - N-1 thru measurements to determine N load match and 2(N-1) transmission tracking terms. The remaining (N-1)x(N-2) transmission tracking terms can be calculated by applying the concatenation rule.
    - Measuring Nx(N-1) crosstalk terms.

Measurement results. Comparison: P2P 12-term vs. 2-port ECal



Measurement results. Comparison: P2P 12-term, 2-port ECal, Full 4P LSQ



Thank you !

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