Investigation of Calibration Methods for Multiport VNA Measurements

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Outline

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S-Parameter Measurements (N-port case)

• Conventional (ideal) S-Parameters consider solely incident signals $a_i$ of source ports for each switch position ($I, II, ..., N$)

$$[S] = \begin{bmatrix} b_1^I & b_1^{II} & \cdots & b_1^N \\ b_2^I & b_2^{II} & \cdots & b_2^N \\ \vdots & \vdots & \ddots & \vdots \\ b_N^I & b_N^{II} & \cdots & b_N^N \end{bmatrix} \begin{bmatrix} a_1^I & 0 & \cdots & 0 \\ 0 & a_2^{II} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_N^N \end{bmatrix}^{-1}$$

• Generalised S-Parameters (denoted by $M$) consider incident signals $m_{ai}$ of all ports for each switch position ($I, II, ..., N$) and removes port match errors (Switch terms)

$$[M] = \begin{bmatrix} m_{b1}^I & m_{b1}^{II} & \cdots & m_{b1}^N \\ m_{b2}^I & m_{b2}^{II} & \cdots & m_{b2}^N \\ \vdots & \vdots & \ddots & \vdots \\ m_{bN}^I & m_{bN}^{II} & \cdots & m_{bN}^N \end{bmatrix} \begin{bmatrix} m_{a1}^I & m_{a1}^{II} & \cdots & m_{a1}^N \\ m_{a2}^I & m_{a2}^{II} & \cdots & m_{a2}^N \\ \vdots & \vdots & \ddots & \vdots \\ m_{aN}^I & m_{aN}^{II} & \cdots & m_{aN}^N \end{bmatrix}^{-1}$$

Fig. 1: Generalised model of a multiport test set
S-Parameter Measurements (2-port case)

• Conventional (ideal) S-Parameters:

\[
[S] = \begin{bmatrix} b_1^I & b_1^{II} \\ b_2^I & b_2^{II} \end{bmatrix} \cdot \begin{bmatrix} a_1^I & 0 \\ 0 & a_2^{II} \end{bmatrix}^{-1} = \begin{bmatrix} b_1^I & b_1^{II} \\ b_2^I & b_2^{II} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ a_1^I & a_2^{II} \end{bmatrix} = \begin{bmatrix} b_1^I & b_1^{II} \\ b_2^I & b_2^{II} \end{bmatrix}
\]

• Generalised S-Parameters:

\[
[M] = \begin{bmatrix} m_1^I & m_1^{II} \\ m_2^I & m_2^{II} \end{bmatrix} \cdot \begin{bmatrix} m_1^I & m_1^{II} \\ m_2^I & m_2^{II} \end{bmatrix}^{-1} = \begin{bmatrix} \frac{m_1^I}{m_1} - \frac{m_1^{II}}{m_1} & m_1^{II} \\ \frac{m_2^I}{m_2} - \frac{m_2^{II}}{m_2} & m_2^{II} \end{bmatrix} = \begin{bmatrix} \frac{m_1^I}{m_1} & m_1^{II} \\ \frac{m_2^I}{m_2} & m_2^{II} \end{bmatrix}
\]

\[
= \begin{bmatrix} \frac{m_1^I}{m_1} - \frac{m_1^{II}}{m_1} & m_1^{II} \\ \frac{m_2^I}{m_2} - \frac{m_2^{II}}{m_2} & m_2^{II} \end{bmatrix} \cdot \begin{bmatrix} \frac{m_1^I}{m_1} & m_1^{II} \\ \frac{m_2^I}{m_2} & m_2^{II} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{\Gamma}_1 & \mathbf{\Gamma}_2 \\ \mathbf{\Gamma}_2 & \mathbf{\Gamma}_1 \end{bmatrix}
\]

with abbreviations:

\[
d = 1 - \frac{m_2^I}{m_1} \cdot \frac{m_2^{II}}{m_1} \mathbf{\Gamma}_1 \mathbf{\Gamma}_2 \quad \text{and switch terms:} \quad \mathbf{\Gamma}_1 = \frac{m_1}{m_2^I}, \quad \mathbf{\Gamma}_2 = \frac{m_1}{m_2^{II}}
\]

• In the ideal case, where \( \mathbf{\Gamma}_1 = \mathbf{\Gamma}_2 = 0 \), \([M]\) reduces to \([S]\).
Full Multiport Least Squares Calibration Method

- Applies a \((4N^2-1)\)-term error model.
- Requires dual reflectometers (couplers) at each test port to measure the generalised S-parameters \((M)\).
- For the sake of clarity the formalism is described for 2-port case:

- The corrected signals \(a\) and \(b\) are related to the raw signals \(m\) by the error term matrix \([C]\) as follows:

\[
\begin{pmatrix}
    b_1 \\
    b_2 \\
    a_1 \\
    a_2
\end{pmatrix} =
\begin{bmatrix}
    m_{a1} \\
    m_{a2} \\
    m_{b1} \\
    m_{b2}
\end{bmatrix} =
\begin{bmatrix}
    [G] & [E] \\
    [F] & [H]
\end{bmatrix}
\begin{pmatrix}
    m_{a1} \\
    m_{a2} \\
    m_{b1} \\
    m_{b2}
\end{pmatrix}
\Rightarrow
\begin{pmatrix}
    b_1 \\
    b_2 \\
    a_1 \\
    a_2
\end{pmatrix} =
\begin{bmatrix}
    G & m_{a1} \\
    E & m_{b1}
\end{bmatrix}
\begin{pmatrix}
    a_1 \\
    a_2
\end{pmatrix} +
\begin{bmatrix}
    F & m_{a1} \\
    H & m_{b1}
\end{bmatrix}
\begin{pmatrix}
    b_1 \\
    b_2
\end{pmatrix}
\tag{1}
\]

\[
\begin{pmatrix}
    b_1 \\
    b_2
\end{pmatrix} =
\begin{bmatrix}
    m_{a1} \\
    m_{a2}
\end{bmatrix} +
\begin{bmatrix}
    m_{b1} \\
    m_{b2}
\end{bmatrix}
\tag{2}
\]

- By substituting Eqs. (1) and (2) into the expression for S-parameters, we get the fundamental equation for each switch position:

\[
\begin{pmatrix}
    b_1 \\
    b_2
\end{pmatrix} =
\begin{bmatrix}
    S & m_{a1} \\
    m_{a2} & m_{b2}
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
    G & m_{a1} \\
    E & m_{b1}
\end{bmatrix}
\begin{pmatrix}
    m_{a1} \\
    m_{a2} \\
    m_{b1} \\
    m_{b2}
\end{pmatrix}
= [S]
\begin{bmatrix}
    F & m_{a1} \\
    H & m_{b1}
\end{bmatrix}
\begin{pmatrix}
    m_{a1} \\
    m_{a2}
\end{pmatrix} +
\begin{bmatrix}
    F & m_{a1} \\
    H & m_{b1}
\end{bmatrix}
\begin{pmatrix}
    m_{b1} \\
    m_{b2}
\end{pmatrix}
\]
Full Multiport Least Squares Calibration Method

• Combining the equations for the two switch positions I and II:

\[
[G] \begin{bmatrix}
  m_{a1}^I & m_{a1}^{II} \\
  m_{a2}^I & m_{a2}^{II}
\end{bmatrix} + [E] \begin{bmatrix}
  m_{b1}^I & m_{b1}^{II} \\
  m_{b2}^I & m_{b2}^{II}
\end{bmatrix} = [S] \begin{bmatrix}
  m_{a1}^I & m_{a1}^{II} \\
  m_{a2}^I & m_{a2}^{II}
\end{bmatrix} + [H] \begin{bmatrix}
  m_{b1}^I & m_{b1}^{II} \\
  m_{b2}^I & m_{b2}^{II}
\end{bmatrix}
\]

and substituting the expression for the generalised S-parameters:

\[
[M] = \begin{bmatrix}
  m_{b1}^I & m_{b1}^{II} \\
  m_{b2}^I & m_{b2}^{II}
\end{bmatrix} \begin{bmatrix}
  m_{a1}^I & m_{a1}^{II} \\
  m_{a2}^I & m_{a2}^{II}
\end{bmatrix}^{-1} = \begin{bmatrix}
  m_{11} & m_{12} \\
  m_{21} & m_{22}
\end{bmatrix}
\]

leads to the final equation:

\[
[G] + [E][M] = [S][F] + [H][M]
\]

(3)

• Equation (3) is applied for
  • calculating error terms from calibration measurements and subsequently
  • calculating S-parameters from DUT measurements.

• Equation (3) can be rearranged to form a linear equation system, solved for the
  error terms [G], [E], [F] and [H].
Full Multiport Least Squares Calibration Method

- Each calibration measurement \((i)\) yields:

\[
-\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix} \begin{bmatrix} m_{11}^i & m_{12}^i \\ m_{21}^i & m_{22}^i \end{bmatrix} + \begin{bmatrix} S_{11}^i & S_{12}^i \\ S_{21}^i & S_{22}^i \end{bmatrix} \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} + \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} m_{11}^i & m_{12}^i \\ m_{21}^i & m_{22}^i \end{bmatrix} = 0
\]

- Evaluating matrix multiplications and sorting the coefficients of error terms in a coefficient matrix leads to:

\[
\begin{bmatrix}
-1 & 0 & 0 & 0 & -m_{11}^i & -m_{21}^i & 0 & 0 & S_{11}^i & 0 & S_{12}^i & 0 & S_{11}^i m_{11}^i & S_{12}^i m_{21}^i & S_{11}^i m_{11}^i & S_{12}^i m_{21}^i \\
0 & -1 & 0 & 0 & -m_{12}^i & -m_{22}^i & 0 & 0 & 0 & S_{11}^i & 0 & S_{12}^i & S_{11}^i m_{12}^i & S_{12}^i m_{22}^i & S_{11}^i m_{12}^i & S_{12}^i m_{22}^i \\
0 & 0 & -1 & 0 & 0 & 0 & -m_{11}^i & -m_{21}^i & S_{21}^i & 0 & S_{22}^i & 0 & S_{21}^i m_{11}^i & S_{22}^i m_{21}^i & S_{21}^i m_{11}^i & S_{22}^i m_{21}^i \\
0 & 0 & 0 & -1 & 0 & 0 & -m_{12}^i & -m_{22}^i & 0 & S_{21}^i & 0 & S_{22}^i & S_{21}^i m_{12}^i & S_{22}^i m_{22}^i & S_{21}^i m_{12}^i & S_{22}^i m_{22}^i
\end{bmatrix}
\begin{bmatrix}
\tilde{e}
\end{bmatrix} = \begin{bmatrix} 0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

(4)

with error term vector:

\[
\tilde{e} = \begin{bmatrix} G_{11}, G_{12}, G_{21}, G_{22}, E_{11}, E_{12}, E_{21}, E_{22}, F_{11}, F_{12}, F_{21}, F_{22}, H_{11}, H_{12}, H_{21}, H_{22} \end{bmatrix}^T
\]

- The homogeneous equation system (4) leads to the trivial zero solution!
Full Multiport Least Squares Calibration Method

- Normalising the error terms by setting $G_{11}$ to 1 (e.g.) leads to the solvable inhomogeneous equation system:

$$
\begin{bmatrix}
0 & 0 & 0 & -m_{11}^i - m_{21}^i & 0 & 0 & S_{11}^i & 0 & S_{12}^i & 0 & S_{11} m_{11}^i & S_{11} m_{21}^i & S_{12} m_{11}^i & S_{12} m_{21}^i \\
-1 & 0 & 0 & -m_{12}^i - m_{22}^i & 0 & 0 & 0 & S_{11}^i & 0 & S_{12}^i & 0 & S_{11} m_{12}^i & S_{11} m_{22}^i & S_{12} m_{12}^i & S_{12} m_{22}^i \\
0 & -1 & 0 & 0 & 0 & -m_{11}^i - m_{21}^i & S_{21}^i & 0 & S_{22}^i & 0 & S_{21} m_{11}^i & S_{21} m_{21}^i & S_{22} m_{11}^i & S_{22} m_{21}^i \\
0 & 0 & -1 & 0 & 0 & -m_{12}^i - m_{22}^i & 0 & S_{21}^i & 0 & S_{22}^i & 0 & S_{21} m_{12}^i & S_{21} m_{22}^i & S_{22} m_{12}^i & S_{22} m_{22}^i
\end{bmatrix}
\begin{bmatrix}
\varepsilon'_{A} \\
\varepsilon'_{B}
\end{bmatrix} =
\begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}
$$

with modified error term vector:

$$
\varepsilon' = (G_{12}, G_{21}, G_{22}, E_{11}, E_{12}, E_{21}, E_{22}, F_{11}, F_{12}, F_{21}, F_{22}, H_{11}, H_{12}, H_{21}, H_{22})^T
$$

- Combining the results of n>=5 calibration measurements leads to the least squares solution for the error terms.

$$
\begin{bmatrix}
A^i \\
\vdots \\
A^n
\end{bmatrix} \varepsilon' =
\begin{bmatrix}
(B^1) \\
\vdots \\
(B^n)
\end{bmatrix}
$$

- Once the error terms have been determined, S-parameters of the DUT can be calculated applying:

$$
[S_{DUT}] = \{G\} + [E][M_{DUT}][F] + [H][M_{DUT}]^{-1}
$$
Four-port Calibration Procedure

- Due to the lack of real 4-port calibration standards, differing standards can be constructed by combining 1-port standards and thru connections:
  - Standards: Open (O), Offset shorts (S1 ... S4), Broadband Load (L)
  - DUT: Power splitter

<table>
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<tr>
<th>Calibration Measurements</th>
<th>Ports</th>
<th>P1 (m)</th>
<th>P3 (m)</th>
<th>P4 (f)</th>
<th>P2 (f)</th>
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<td>O</td>
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<td>S2</td>
<td>S3</td>
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<td>PS\textsubscript{out} \text{R}</td>
<td>S1</td>
<td>PS\textsubscript{in}</td>
<td></td>
</tr>
</tbody>
</table>
Measurement results. Comparison: Full 4-port LSQ vs. 2-port ECAL
Port-to-Port Multiport Calibration Procedure

- Applies common 2-port 12-term error model:

  ![Diagram](image)

  - Forward direction
  - Reverse direction

- Relies on successive determination of error terms with the help of appropriate calibration measurements:
  - N 1-port calibrations to determine directivity, source match and reflection tracking terms for each port.
  - N-1 thru measurements to determine N load match and 2(N-1) transmission tracking terms. The remaining (N-1)x(N-2) transmission tracking terms can be calculated by applying the concatenation rule.
  - Measuring Nx(N-1) crosstalk terms.
Measurement results. Comparison: P2P 12-term vs. 2-port ECaL
Measurement results. Comparison: P2P 12-term, 2-port ECal, Full 4P LSQ
Thank you!

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