Investigating Connection Repeatability of Waveguide Devices at Frequencies from 750 GHz to 1.1 THz

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HF-Circuits EMRP Project – 1st Workshop & Euro-ANAMET meeting, 5th December 2013

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Overview

- Background
- Experimental Method
- Data Analysis
- Results
- Observations
- Uncertainty Considerations
- Summary

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Background

- In recent years, the upper frequency limit of VNAs has increased to 1.1 THz
- Most VNAs above 110 GHz use waveguide as the test ports
- The size of waveguide above 110 GHz is often very small (e.g. 250 μm x 125 μm at 1.1 THz)
- Connection alignment for these small waveguides can cause large measurement errors

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Background (contd)

- Connection repeatability of small waveguides introduces random errors into the VNA measurements
- This work investigates the likely size of these errors for measurements in WM-250 (WR-01) from 750 GHz to 1.1 THz
- A framework is also described for accommodating these errors into an overall uncertainty budget for VNAs operating from 750 GHz to 1.1 THz and other submillimeter-wave frequency bands

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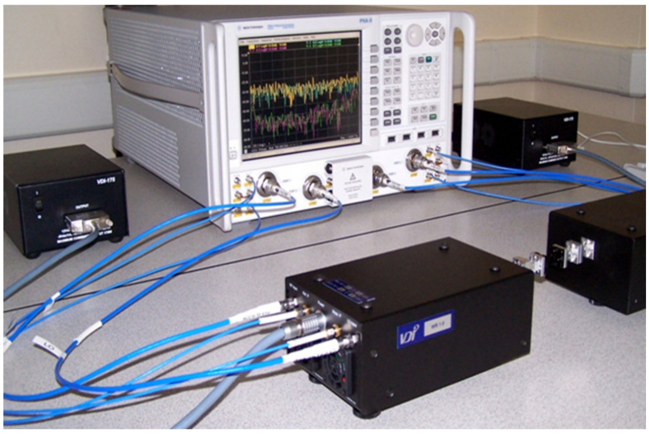
Experimental Method

Agilent PNA-X VNA with VDI Extender Heads: 750 GHz to 1100 GHz

Situated at:
'The Roger Pollard High Frequency Measurements Laboratory'

University of Leeds
UK



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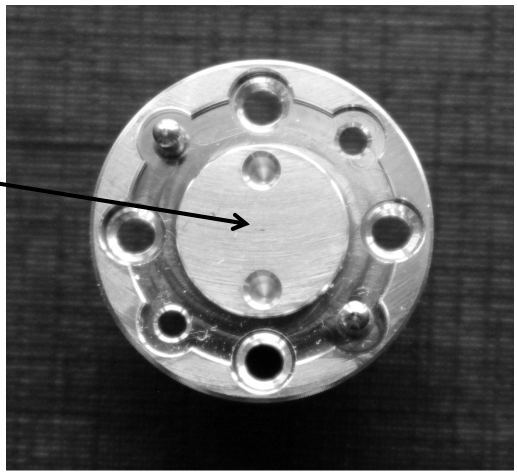
Experimental Method (contd)

VNA Test Ports

Aperture:
- WM-250 (WR-01)
  250 µm × 125 µm

Flange/interface:
- ¾″ round (UG-387)
- 4 outer alignment pins used for flange alignment



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Experimental Method (contd)

Test conditions:

- Input power level ≈ -35 dBm (0.3 μW)
- IF bandwidth: 30 Hz
- Averaging factor = 1 (i.e. no numerical averaging)

- Frequency range: 750 GHz to 1100 GHz
- Frequency step size: 1.75 GHz (i.e. 201 points)

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Experimental Method (contd)

Calibration:

- One-port calibration, using three 'known' loads
- Flush Short / Offset Short / Near-matched load

Repeatability:

- Devices: the same three calibration standards
- 12 disconnect/reconnects for each device
- No flange inversion between disconnect/reconnects

Measurand:

- Complex-valued linear reflection coefficient

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Data analysis

We use the experimental standard deviation as the measure of variability in the observed repeatability values

Let Γ be the complex-valued linear reflection coefficient written in terms of its real, Γ_R, and imaginary, Γ_I, components:

$$\Gamma = \Gamma_R + j\,\Gamma_I$$

(with $j^2 = -1$)

We make n repeated determinations of Γ (in our case, we have chosen $n = 12$)

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Data analysis (contd)

Real component, Γ_R

Experimental variance:

$$s^2(\Gamma_{R_i}) = \frac{1}{n-1}\sum_{k=1}^{n}(\Gamma_{R_k} - \overline{\Gamma}_R)^2$$

where

$$\overline{\Gamma}_R = \frac{1}{n}\sum_{i=1}^{n}\Gamma_{R_i}$$

Experimental standard deviation, $s(\Gamma_{R_i})$, is the positive square root of $s^2(\Gamma_{R_i})$

Imaginary component, Γ_I

Experimental variance:

$$s^2(\Gamma_{I_i}) = \frac{1}{n-1}\sum_{j=1}^{n}(\Gamma_{I_j} - \overline{\Gamma}_I)^2$$

where

$$\overline{\Gamma}_I = \frac{1}{n}\sum_{i=1}^{n}\Gamma_{I_i}$$

Experimental standard deviation, $s(\Gamma_{I_i})$, is the positive square root of $s^2(\Gamma_{I_i})$

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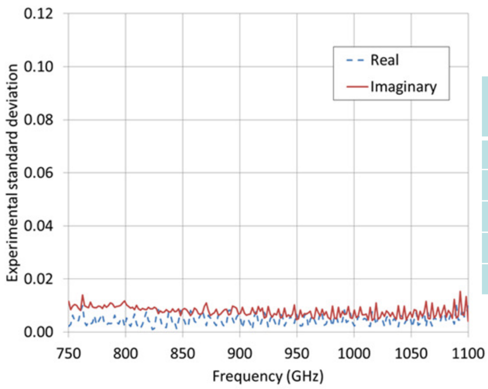
Results – Near-matched load



Frequency (GHz)	$s(\Gamma_{R_i})$	$s(\Gamma_{I_i})$
750.0	0.002 2	0.011 5
837.5	0.007 5	0.007 6
925.0	0.001 8	0.009 7
1012.5	0.002 5	0.004 5
1100.0	0.010 5	0.004 1

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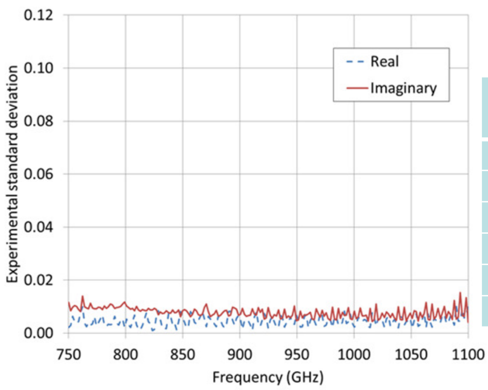
Results – Near-matched load



Frequency (GHz)	$s(\Gamma_{R_i})$	$s(\Gamma_{I_i})$
750.0	0.002 2	0.011 5
837.5	0.007 5	0.007 6
925.0	0.001 8	0.009 7
1012.5	0.002 5	0.004 5
1100.0	0.010 5	0.004 1
Average	0.004 6	0.007 8

$s(\Gamma_I) \approx 2 \times s(\Gamma_R)$

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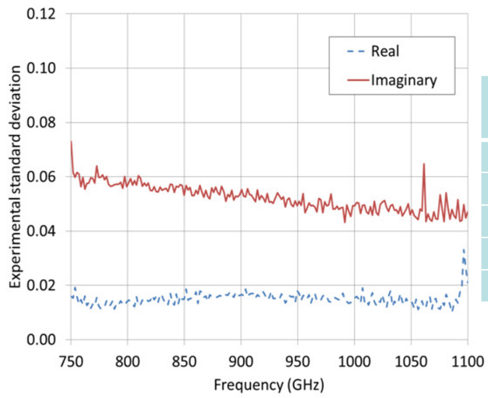
Results – Flush short



Frequency (GHz)	$s(\Gamma_{R_i})$	$s(\Gamma_{I_i})$
750.0	0.015 9	0.072 9
837.5	0.017 0	0.057 2
925.0	0.015 4	0.050 6
1012.5	0.012 9	0.046 3
1100.0	0.021 0	0.047 1

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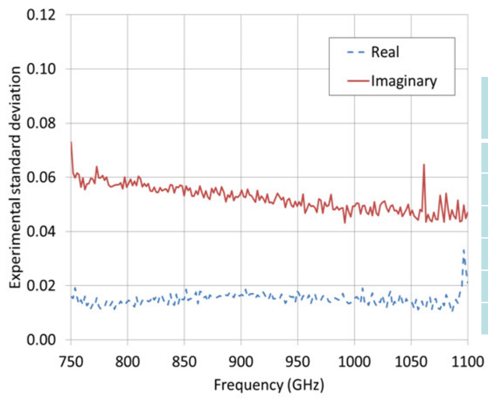
Results – Flush short



Frequency (GHz)	$s(\Gamma_{R_i})$	$s(\Gamma_{I_i})$
750.0	0.015 9	0.072 9
837.5	0.017 0	0.057 2
925.0	0.015 4	0.050 6
1012.5	0.012 9	0.046 3
1100.0	0.021 0	0.047 1
Average	**0.015 0**	**0.052 3**

$s(\Gamma_I) \approx 3.5 \times s(\Gamma_R)$

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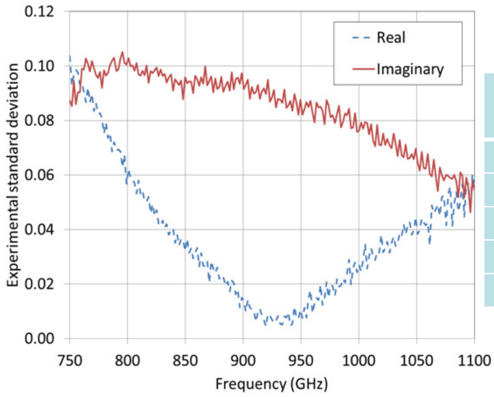
Results – Offset short



Frequency (GHz)	$s(\Gamma_{R_i})$	$s(\Gamma_{I_i})$
750.0	0.103 6	0.087 3
837.5	0.039 6	0.096 9
925.0	0.006 9	0.088 5
1012.5	0.030 3	0.072 5
1100.0	0.056 1	0.054 6

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Observations

Device	Nominal reflection coefficient, Γ	$s(\Gamma_R)$	$s(\Gamma_I)$	$s(\Gamma_R)$ & $s(\Gamma_I)$
Near-matched load	$\|\Gamma\| \approx 0$	0.005	0.008	$s(\Gamma_I) \approx 2 \times s(\Gamma_R)$
Flush short	$\|\Gamma\| \approx 1$ phase = ±180°	0.015	0.052	$s(\Gamma_I) \approx 3.5 \times s(\Gamma_R)$
Offset short	$\|\Gamma\| \approx 1$ phase varies with frequency	0.01→0.10	0.05→0.10	$s(\Gamma_I)$ and $s(\Gamma_R)$ vary with frequency

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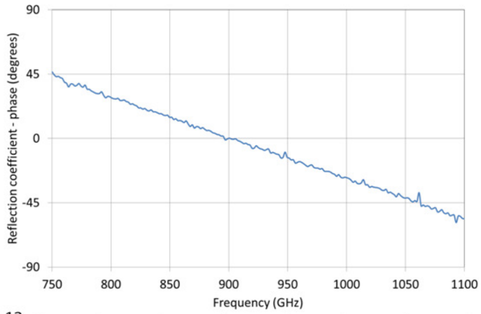
Observations (contd)

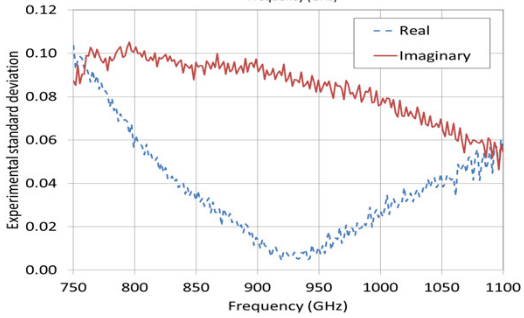
Offset short: phase, φ

750 GHz: φ ≈ +45 °

900 GHz: φ ≈ 0 °

1100 GHz: φ ≈ −55 °





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Observations (contd)

Repeatability can be represented as regions (i.e. areas) in the complex Γ-plane

$s(\Gamma_R) \neq s(\Gamma_I)$ suggests regions that are elliptical

The covariance of Γ_R and Γ_I is needed to fully define the ellipse:

$$s(\Gamma_R, \Gamma_I) = \frac{1}{(n-1)} \sum_{k=1}^{n} \left(\Gamma_{R_k} - \bar{\Gamma}_R\right)\left(\Gamma_{I_k} - \bar{\Gamma}_I\right)$$

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Observations (contd)

The ellipse can be calculated using the covariance matrix:

$$V = \begin{pmatrix} s^2(\Gamma_R) & s(\Gamma_R, \Gamma_I) \\ s(\Gamma_I, \Gamma_R) & s^2(\Gamma_I) \end{pmatrix}$$

The correlation coefficient can be calculated to indicate the degree of correlation between Γ_R and Γ_I:

$$r(\Gamma_R, \Gamma_I) = \frac{s(\Gamma_R, \Gamma_I)}{s(\Gamma_R)s(\Gamma_I)}$$

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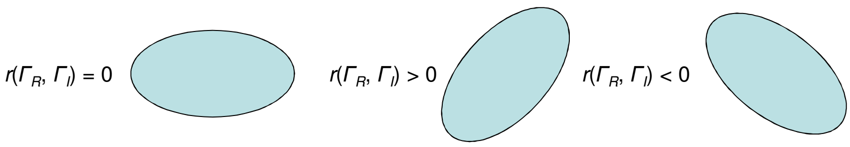
Observations (contd)

A correlation coefficient, $r(\Gamma_R, \Gamma_I)$, can have a value anywhere within the range:

$$-1 \leq r(\Gamma_R, \Gamma_I) \leq +1$$

The value of $r(\Gamma_R, \Gamma_I)$ affects the orientation of the ellipse:



The correlation coefficient helps interpret the data

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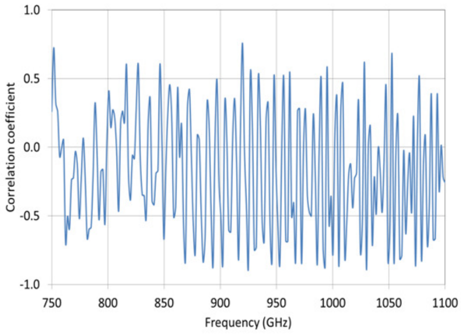
Observations: Near-matched load

$s(\Gamma_I) \approx 2 \times s(\Gamma_R)$

Repeatability ellipses



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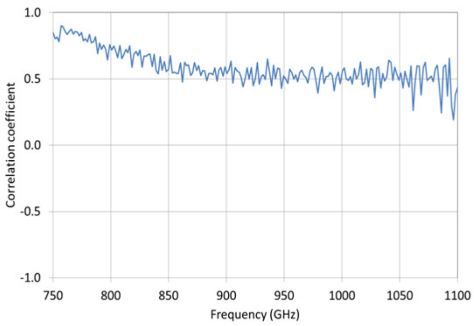
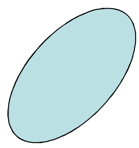
Observations: Flush short

$s(\Gamma_I) \approx 3.5 \times s(\Gamma_R)$

Repeatability ellipses



$r(\Gamma_R, \Gamma_I) > 0$



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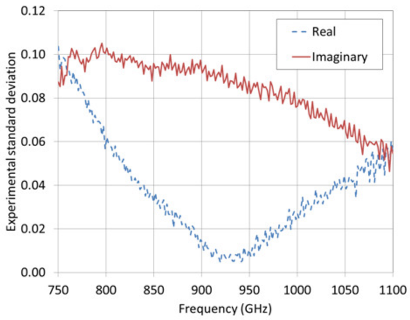
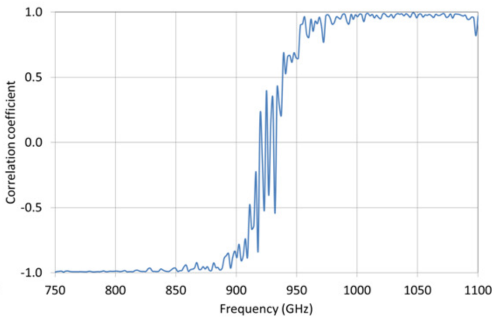
Observations: Offset short

Repeatability ellipses





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Uncertainty considerations

The generalized measurement model:

$$\boldsymbol{y} = f(\boldsymbol{x})$$

Applied to our situation, becomes:

$$\Gamma_M = f(\Gamma_1, \Gamma_2, \Gamma_3)$$

The uncertainty can be represented as:

$$V_y = J \; V_x \, J^T$$

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$$V_y = J\ V_x\ J^T$$

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Uncertainty considerations - contd

$$V_y = \begin{pmatrix} u^2(\Gamma_{M_R}) & u(\Gamma_{M_R}, \Gamma_{M_I}) \\ u(\Gamma_{M_I}, \Gamma_{M_R}) & u^2(\Gamma_{M_I}) \end{pmatrix}$$

$$J = \begin{pmatrix} \partial f_R/\partial\Gamma_{1_R} & \partial f_R/\partial\Gamma_{1_I} & \partial f_R/\partial\Gamma_{2_R} & \partial f_R/\partial\Gamma_{2_I} & \partial f_R/\partial\Gamma_{3_R} & \partial f_R/\partial\Gamma_{3_I} \\ \partial f_I/\partial\Gamma_{1_R} & \partial f_I/\partial\Gamma_{1_I} & \partial f_I/\partial\Gamma_{2_R} & \partial f_I/\partial\Gamma_{2_I} & \partial f_I/\partial\Gamma_{3_R} & \partial f_I/\partial\Gamma_{3_I} \end{pmatrix}$$

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$$V_y = J\ V_x\ J^T$$

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Uncertainty considerations - contd

$$V_x = \begin{pmatrix} u^2(\Gamma_{1_R}) & u(\Gamma_{1_R}, \Gamma_{1_I}) & u(\Gamma_{1_R}, \Gamma_{2_R}) & u(\Gamma_{1_R}, \Gamma_{2_I}) & u(\Gamma_{1_R}, \Gamma_{3_R}) & u(\Gamma_{1_R}, \Gamma_{3_I}) \\ u(\Gamma_{1_I}, \Gamma_{1_R}) & u^2(\Gamma_{1_I}) & u(\Gamma_{1_I}, \Gamma_{2_R}) & u(\Gamma_{1_I}, \Gamma_{2_I}) & u(\Gamma_{1_I}, \Gamma_{3_R}) & u(\Gamma_{1_I}, \Gamma_{3_I}) \\ u(\Gamma_{2_R}, \Gamma_{1_R}) & u(\Gamma_{2_R}, \Gamma_{1_I}) & u^2(\Gamma_{2_R}) & u(\Gamma_{2_R}, \Gamma_{2_I}) & u(\Gamma_{2_R}, \Gamma_{3_R}) & u(\Gamma_{2_R}, \Gamma_{3_I}) \\ u(\Gamma_{2_I}, \Gamma_{1_R}) & u(\Gamma_{2_I}, \Gamma_{1_I}) & u(\Gamma_{2_I}, \Gamma_{2_R}) & u^2(\Gamma_{2_I}) & u(\Gamma_{2_I}, \Gamma_{3_R}) & u(\Gamma_{2_I}, \Gamma_{3_I}) \\ u(\Gamma_{3_R}, \Gamma_{1_R}) & u(\Gamma_{3_R}, \Gamma_{1_I}) & u(\Gamma_{3_R}, \Gamma_{2_R}) & u(\Gamma_{3_R}, \Gamma_{2_I}) & u^2(\Gamma_{3_R}) & u(\Gamma_{3_R}, \Gamma_{3_I}) \\ u(\Gamma_{3_I}, \Gamma_{1_R}) & u(\Gamma_{3_I}, \Gamma_{1_I}) & u(\Gamma_{3_I}, \Gamma_{2_R}) & u(\Gamma_{3_I}, \Gamma_{2_I}) & u(\Gamma_{3_I}, \Gamma_{3_R}) & u^2(\Gamma_{3_I}) \end{pmatrix}$$

$$V_x = \begin{pmatrix} V_{\Gamma_1} & 0 & 0 \\ 0 & V_{\Gamma_2} & 0 \\ 0 & 0 & V_{\Gamma_3} \end{pmatrix}$$

where $V_{\Gamma_1} = \begin{pmatrix} u^2(\Gamma_{1_R}) & u(\Gamma_{1_R}, \Gamma_{1_I}) \\ u(\Gamma_{1_I}, \Gamma_{1_R}) & u^2(\Gamma_{1_I}) \end{pmatrix}$ etc

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Summary

- Measurement repeatability in WM-250 (750 GHz to 1100 GHz)
- Data analyzed in terms of Real & Imaginary components
- Repeatability depends on the device – 'worst' for offset short
- Data visualization (i.e. repeatability ellipses) used to facilitate understanding of the measurement error mechanisms
- Application to uncertainty evaluation techniques
- General method – applicable to other waveguide bands

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