Investigating Connection Repeatability of Waveguide Devices at Frequencies from 750 GHz to 1.1 THz

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Overview

- Background
- Experimental Method
- Data Analysis
- Results
- Observations
- Uncertainty Considerations
- Summary
Background

- In recent years, the upper frequency limit of VNAs has increased to 1.1 THz
- Most VNAs above 110 GHz use waveguide as the test ports
- The size of waveguide above 110 GHz is often very small (e.g. 250 µm x 125 µm at 1.1 THz)
- Connection alignment for these small waveguides can cause large measurement errors

Background (contd)

- Connection repeatability of small waveguides introduces random errors into the VNA measurements
- This work investigates the likely size of these errors for measurements in WM-250 (WR-01) from 750 GHz to 1.1 THz
- A framework is also described for accommodating these errors into an overall uncertainty budget for VNAs operating from 750 GHz to 1.1 THz and other submillimeter-wave frequency bands
Experimental Method

Situated at:
The Roger Pollard High Frequency Measurements Laboratory
University of Leeds
UK

Agilent PNA-X VNA with VDI Extender Heads: 750 GHz to 1100 GHz

Experimental Method (contd)

VNA Test Ports

Aperture:
- WM-250 (WR-01)
  250 \( \mu m \times 125 \mu m \)

Flange/interface:
- ¾” round (UG-387)
- 4 outer alignment pins used for flange alignment
Experimental Method (contd)

Test conditions:
- Input power level \(\approx -35 \text{ dBm} \) (0.3 \(\mu\text{W}\))
- IF bandwidth: 30 Hz
- Averaging factor = 1 (i.e. no numerical averaging)
- Frequency range: 750 GHz to 1100 GHz
- Frequency step size: 1.75 GHz (i.e. 201 points)

Calibration:
- One-port calibration, using three ‘known’ loads
- Flush Short / Offset Short / Near-matched load

Repeatability:
- Devices: the same three calibration standards
- 12 disconnect/reconnects for each device
- No flange inversion between disconnect/reconnects

Measurand:
- Complex-valued linear reflection coefficient
Data analysis

We use the experimental standard deviation as the measure of variability in the observed repeatability values.

Let $\Gamma$ be the complex-valued linear reflection coefficient written in terms of its real, $\Gamma_R$, and imaginary, $\Gamma_I$, components:

$$\Gamma = \Gamma_R + j \Gamma_I$$

(with $j^2 = -1$)

We make $n$ repeated determinations of $\Gamma$ (in our case, we have chosen $n = 12$)

Data analysis (contd)

Real component, $\Gamma_R$

Experimental variance:

$$s^2(\Gamma_R) = \frac{1}{n-1} \sum_{k=1}^{n} (\Gamma_{R,k} - \overline{\Gamma}_R)^2$$

where

$$\overline{\Gamma}_R = \frac{1}{n} \sum_{i=1}^{n} \Gamma_{R,i}$$

Experimental standard deviation, $s(\Gamma_R)$, is the positive square root of $s^2(\Gamma_R)$

Imaginary component, $\Gamma_I$

Experimental variance:

$$s^2(\Gamma_I) = \frac{1}{n-1} \sum_{j=1}^{n} (\Gamma_{I,j} - \overline{\Gamma}_I)^2$$

where

$$\overline{\Gamma}_I = \frac{1}{n} \sum_{i=1}^{n} \Gamma_{I,i}$$

Experimental standard deviation, $s(\Gamma_I)$, is the positive square root of $s^2(\Gamma_I)$
Results – Near-matched load

![Graph showing experimental standard deviation vs frequency (GHz)]

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>( \sigma_{\Re} )</th>
<th>( \sigma_{\Im} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>750.0</td>
<td>0.002 2</td>
<td>0.011 5</td>
</tr>
<tr>
<td>837.5</td>
<td>0.007 5</td>
<td>0.007 6</td>
</tr>
<tr>
<td>925.0</td>
<td>0.001 8</td>
<td>0.009 7</td>
</tr>
<tr>
<td>1012.5</td>
<td>0.002 5</td>
<td>0.004 5</td>
</tr>
<tr>
<td>1100.0</td>
<td>0.010 5</td>
<td>0.004 1</td>
</tr>
</tbody>
</table>

Average: \( \sigma_{\Re} = 0.004 6 \), \( \sigma_{\Im} = 0.007 8 \)

\[ s(\Gamma) = 2 \times s(\Gamma_R) \]
Results – Flush short

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>$s(\Gamma_R)$</th>
<th>$s(\Gamma_I)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>750.0</td>
<td>0.015 9</td>
<td>0.072 9</td>
</tr>
<tr>
<td>837.5</td>
<td>0.017 0</td>
<td>0.057 2</td>
</tr>
<tr>
<td>925.0</td>
<td>0.015 4</td>
<td>0.050 6</td>
</tr>
<tr>
<td>1012.5</td>
<td>0.012 9</td>
<td>0.046 3</td>
</tr>
<tr>
<td>1100.0</td>
<td>0.021 0</td>
<td>0.047 1</td>
</tr>
<tr>
<td>Average</td>
<td>0.015 0</td>
<td>0.052 3</td>
</tr>
</tbody>
</table>

$s(\Gamma_I) = 3.5 \times s(\Gamma_R)$
### Results – Offset short

![Graph showing frequency and reflection coefficient](image)

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>$s(\Gamma_R)$</th>
<th>$s(\Gamma_I)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>750.0</td>
<td>0.103</td>
<td>0.087</td>
</tr>
<tr>
<td>837.5</td>
<td>0.039</td>
<td>0.096</td>
</tr>
<tr>
<td>925.0</td>
<td>0.006</td>
<td>0.088</td>
</tr>
<tr>
<td>1012.5</td>
<td>0.000</td>
<td>0.072</td>
</tr>
<tr>
<td>1100.0</td>
<td>0.056</td>
<td>0.054</td>
</tr>
</tbody>
</table>

### Observations

<table>
<thead>
<tr>
<th>Device</th>
<th>Nominal reflection coefficient, $\Gamma$</th>
<th>$s(\Gamma_R)$</th>
<th>$s(\Gamma_I)$</th>
<th>$s(\Gamma_R)$ &amp; $s(\Gamma_I)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Near-matched load</td>
<td>$</td>
<td>\Gamma</td>
<td>= 0$</td>
<td>0.005</td>
</tr>
<tr>
<td>Flush short</td>
<td>$</td>
<td>\Gamma</td>
<td>= 1$ phase $= \pm 180^\circ$</td>
<td>0.015</td>
</tr>
<tr>
<td>Offset short</td>
<td>$</td>
<td>\Gamma</td>
<td>= 1$ phase varies with frequency</td>
<td>0.01→0.10</td>
</tr>
</tbody>
</table>
Observations (contd)

Offset short: phase, $\phi$

750 GHz: $\phi = +45^\circ$

900 GHz: $\phi = 0^\circ$

1100 GHz: $\phi = -55^\circ$

Repeatability can be represented as regions (i.e. areas) in the complex $\Gamma$-plane

$s(\Gamma_R) \neq s(\Gamma_I)$ suggests regions that are elliptical

The covariance of $\Gamma_R$ and $\Gamma_I$ is needed to fully define the ellipse:

$$s(\Gamma_R, \Gamma_I) = \frac{1}{n-1} \sum_{k=1}^{n} (\Gamma_{R_k} - \bar{\Gamma}_R)(\Gamma_{I_k} - \bar{\Gamma}_I)$$
Observations (contd)

The ellipse can be calculated using the covariance matrix:

\[
V = \begin{pmatrix}
    s^2(\Gamma_R) & s(\Gamma_R, \Gamma_I) \\
    s(\Gamma_I, \Gamma_R) & s^2(\Gamma_I)
\end{pmatrix}
\]

The correlation coefficient can be calculated to indicate the degree of correlation between \(\Gamma_R\) and \(\Gamma_I\):

\[
r(\Gamma_R, \Gamma_I) = \frac{s(\Gamma_R, \Gamma_I)}{s(\Gamma_R)s(\Gamma_I)}
\]

Observations (contd)

A correlation coefficient, \(r(\Gamma_R, \Gamma_I)\), can have a value anywhere within the range:

\[-1 \leq r(\Gamma_R, \Gamma_I) \leq +1\]

The value of \(r(\Gamma_R, \Gamma_I)\) affects the orientation of the ellipse:

\[
r(\Gamma_R, \Gamma_I) = 0 \quad r(\Gamma_R, \Gamma_I) > 0 \quad r(\Gamma_R, \Gamma_I) < 0
\]

The correlation coefficient helps interpret the data.
Observations: Near-matched load

Repeatability ellipses

\[ s(\Gamma_I) = 2 \times s(\Gamma_R) \]

Observations: Flush short

Repeatability ellipses

\[ s(\Gamma_I) = 3.5 \times s(\Gamma_R) \]

Correlation coefficient

\[ r(\Gamma_R, \Gamma_I) > 0 \]
Observations: Offset short

Repeatability ellipses

Uncertainty considerations

The generalized measurement model:

\[ y = f(x) \]

Applied to our situation, becomes:

\[ \Gamma_M = f(\Gamma_1, \Gamma_2, \Gamma_3) \]

The uncertainty can be represented as:

\[ V_y = J V_z J^T \]
\[ \mathbf{V}_y = \mathbf{J} \mathbf{V}_x \mathbf{J}^T \]

Uncertainty considerations - contd

\[ \mathbf{V}_y = \begin{pmatrix}
    u^2(f_{M_R}) & u(f_{M_R}, f_{M_I}) \\
    u(f_{M_R}, f_{M_I}) & u^2(f_{M_I})
\end{pmatrix} \]

\[ \mathbf{J} = \begin{pmatrix}
    \frac{\partial f_R}{\partial r_{1_R}} & \frac{\partial f_R}{\partial r_{1_I}} & \frac{\partial f_R}{\partial r_{2_R}} & \frac{\partial f_R}{\partial r_{2_I}} & \frac{\partial f_R}{\partial r_{3_R}} & \frac{\partial f_R}{\partial r_{3_I}} \\
    \frac{\partial f_I}{\partial r_{1_R}} & \frac{\partial f_I}{\partial r_{1_I}} & \frac{\partial f_I}{\partial r_{2_R}} & \frac{\partial f_I}{\partial r_{2_I}} & \frac{\partial f_I}{\partial r_{3_R}} & \frac{\partial f_I}{\partial r_{3_I}}
\end{pmatrix} \]

\[ \mathbf{V}_x = \begin{pmatrix}
    u^2(f_{I_R}) & u(f_{I_R}, f_{I_1}) & u(f_{I_R}, f_{I_2}) & u(f_{I_R}, f_{I_3}) & u(f_{I_R}, f_{I_4}) & u(f_{I_R}, f_{I_5}) \\
    u(f_{I_R}, f_{I_1}) & u^2(f_{I_1}) & u(f_{I_1}, f_{I_2}) & u(f_{I_1}, f_{I_3}) & u(f_{I_1}, f_{I_4}) & u(f_{I_1}, f_{I_5}) \\
    u(f_{I_R}, f_{I_2}) & u(f_{I_1}, f_{I_2}) & u^2(f_{I_2}) & u(f_{I_2}, f_{I_3}) & u(f_{I_2}, f_{I_4}) & u(f_{I_2}, f_{I_5}) \\
    u(f_{I_R}, f_{I_3}) & u(f_{I_1}, f_{I_3}) & u(f_{I_2}, f_{I_3}) & u^2(f_{I_3}) & u(f_{I_3}, f_{I_4}) & u(f_{I_3}, f_{I_5}) \\
    u(f_{I_R}, f_{I_4}) & u(f_{I_1}, f_{I_4}) & u(f_{I_2}, f_{I_4}) & u(f_{I_3}, f_{I_4}) & u^2(f_{I_4}) & u(f_{I_4}, f_{I_5}) \\
    u(f_{I_R}, f_{I_5}) & u(f_{I_1}, f_{I_5}) & u(f_{I_2}, f_{I_5}) & u(f_{I_3}, f_{I_5}) & u(f_{I_4}, f_{I_5}) & u^2(f_{I_5})
\end{pmatrix} \]

\[ \mathbf{V}_x = \begin{pmatrix}
    V_{r_1} & 0 & 0 \\
    0 & V_{r_2} & 0 \\
    0 & 0 & V_{r_3}
\end{pmatrix} \]

where \[ V_{r_1} = \begin{pmatrix}
    u^2(f_{r_1}) & u(f_{r_1}, f_{r_1}) \\
    u(f_{r_1}, f_{r_1}) & u^2(f_{r_1})
\end{pmatrix} \] etc
Summary

- Measurement repeatability in WM-250 (750 GHz to 1100 GHz)
- Data analyzed in terms of Real & Imaginary components
- Repeatability depends on the device – ‘worst’ for offset short
- Data visualization (i.e. repeatability ellipses) used to facilitate understanding of the measurement error mechanisms
- Application to uncertainty evaluation techniques
- General method – applicable to other waveguide bands

Acknowledgement

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