Uncertainty in two-port electronic calibration units: A progress report on the uncertainty model

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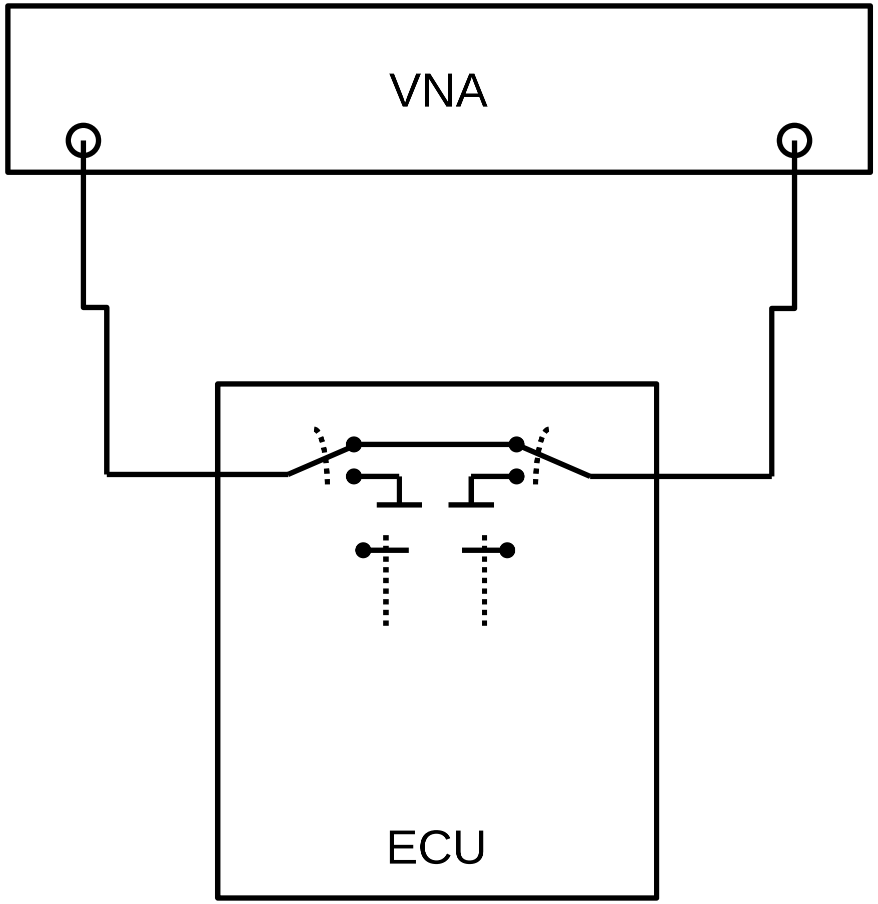
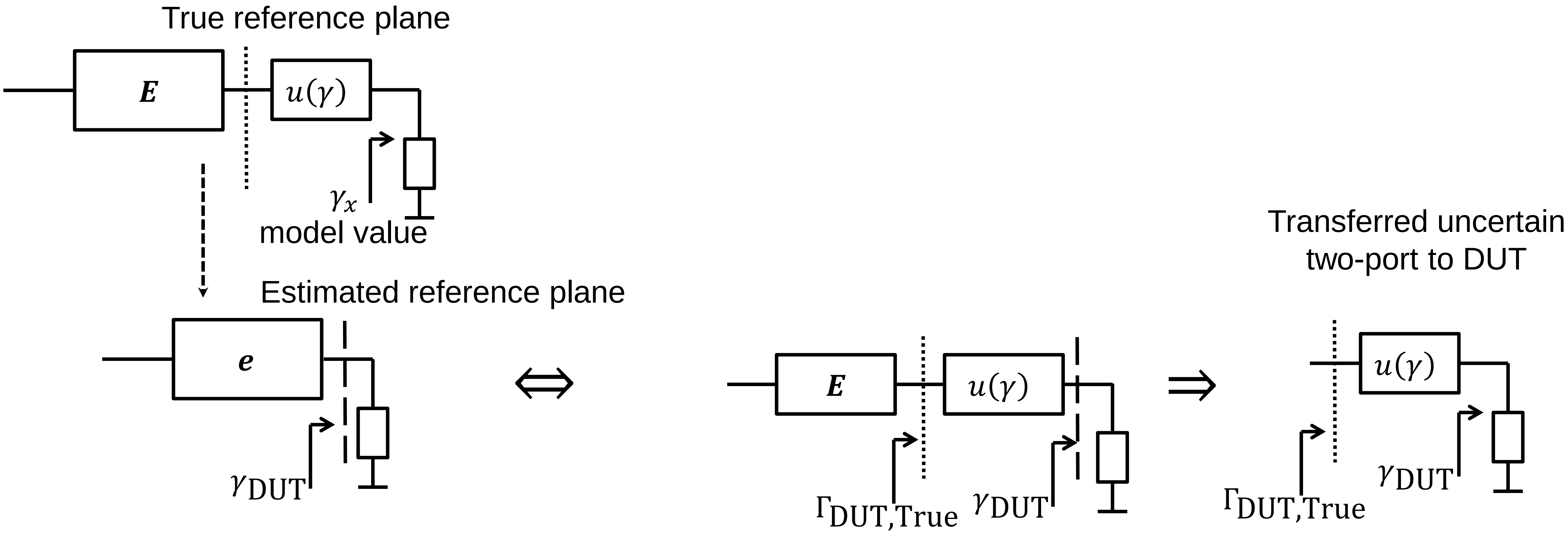
Electronic calibration unit (ECU)

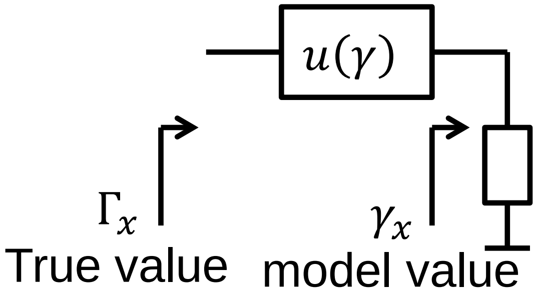


- ECU automatically presents known one- and two-port states to the VNA during calibration
- Uncertainty model for ECU usage and characterization
- Vendor specific
- Uncertainty in states can be propagated to measurements of DUT
  - Uncertainty of each state
  - Modify calibration algorithm
  - Difficult to add for end-user
  - Incompatible with the Euramet guide

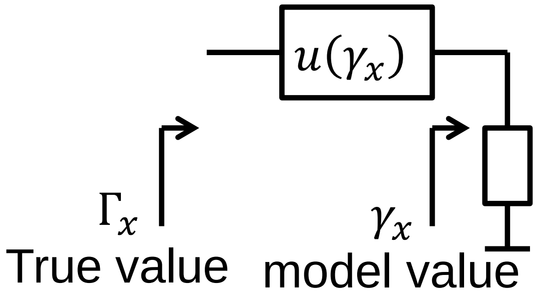
Principle of calibration kit method



Equivalence of calibration kit model and calibration standard model



$$u(\gamma) = \begin{pmatrix} D & \sqrt{1+T} \\ \sqrt{1+T} & M \end{pmatrix}$$



Calibration kit error model

$$\Gamma_s = D + \frac{\gamma_s(1+T)}{1-M\gamma_s}$$
$$\Gamma_o = D + \frac{\gamma_o(1+T)}{1-M\gamma_o}$$
$$\Gamma_l = D + \frac{\gamma_l(1+T)}{1-M\gamma_l}$$

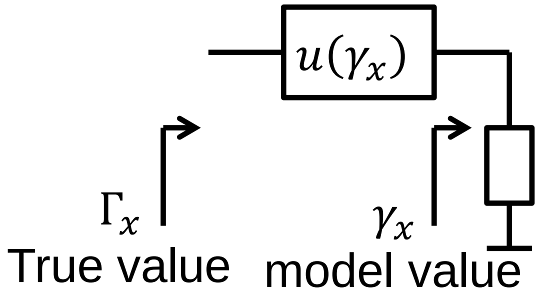
Calibration standard model

$$\Gamma_s = \gamma_s + \delta\gamma_s$$
$$\Gamma_o = \gamma_o + \delta\gamma_o$$
$$\Gamma_l = \gamma_l + \delta\gamma_l$$

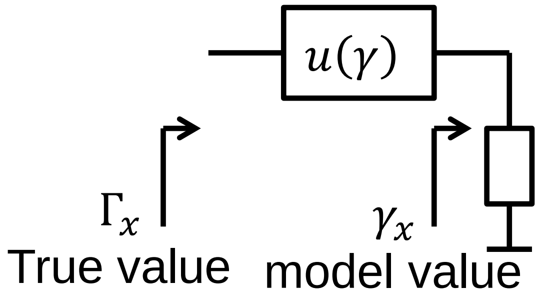
$$\begin{cases} \delta\gamma_s = D + \dfrac{\gamma_s(M\gamma_s + T)}{1-M\gamma_s} \\ \delta\gamma_o = D + \dfrac{\gamma_o(M\gamma_o + T)}{1-M\gamma_o} \\ \delta\gamma_l = D + \dfrac{\gamma_l(M\gamma_l + T)}{1-M\gamma_l} \end{cases}$$

Uncertainty computation

- From uncertainties associated with the standards
  - Propagate uncertainties through the calibration algorithm
  - Need access to details of the algorithm
  - Most flexible approach



- From calibration kit uncertainties
  - Based on methods from the SOLT/SOLR family
  - Derived uncertainty equations for both known and unknown thru



$$u(\gamma) = \begin{pmatrix} \dfrac{D}{\sqrt{1+T}} & \dfrac{\sqrt{1+T}}{M} \end{pmatrix}$$

SOLT/SOLR Linearized uncertainty model

- Use calibration kit model for the one-port standards
- Nominally ideal Thru

SOLT

- $S_{11} \approx s_{11} + (1 - s_{12}s_{21})D_1 + s_{11}^2 M_1 + s_{11}T_1 + s_{12}s_{21}S_{t,11}$
- $S_{21} \approx s_{21}(s_{11}M_1 - s_{22}D_1 + s_{22}S_{t,11} + S_{t,21})$

SOLR

- $S_{11} \approx s_{11} + D_1 + s_{11}^2 M_1 + s_{12}s_{21}M_2 + s_{11}T_1$
- $S_{21} \approx s_{21}(1 + s_{11}M_1 + s_{22}M_2 + \frac{T_1+T_2}{2} + \frac{S_{t,12}-S_{t,21}}{2})$

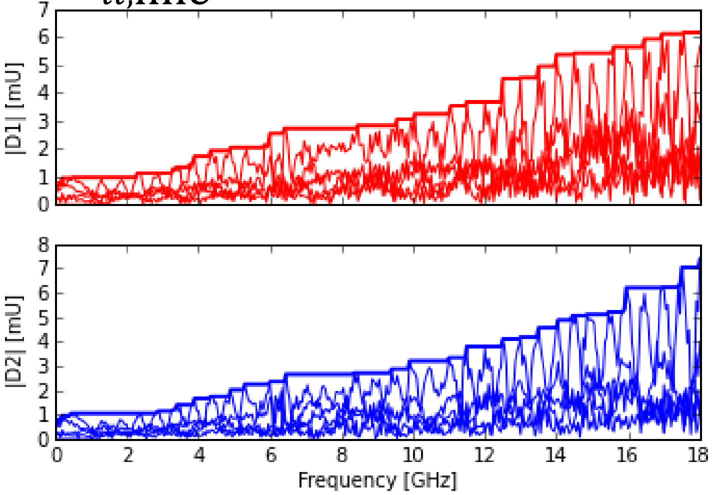
Zero for reciprocal Thru standard

$S_{t,ij}$ S-parameters of thru standard for desired system impedance

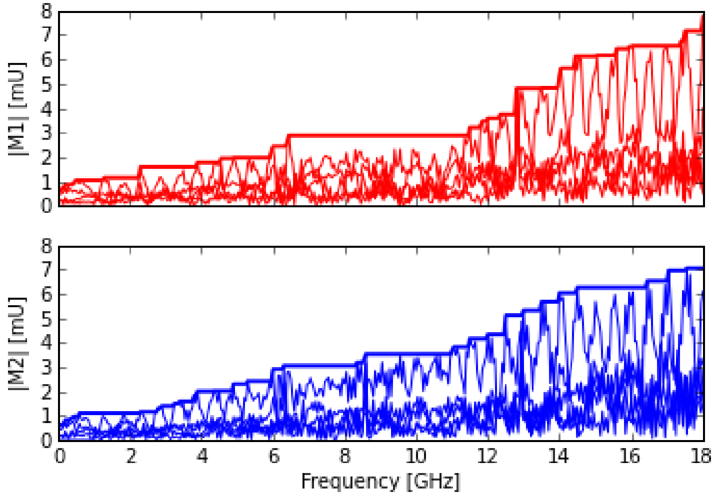
Two-port ripple technique

- Estimating D, T, M from air-lines
- Measure thru and line standards using SOLT and SOLR calibrations

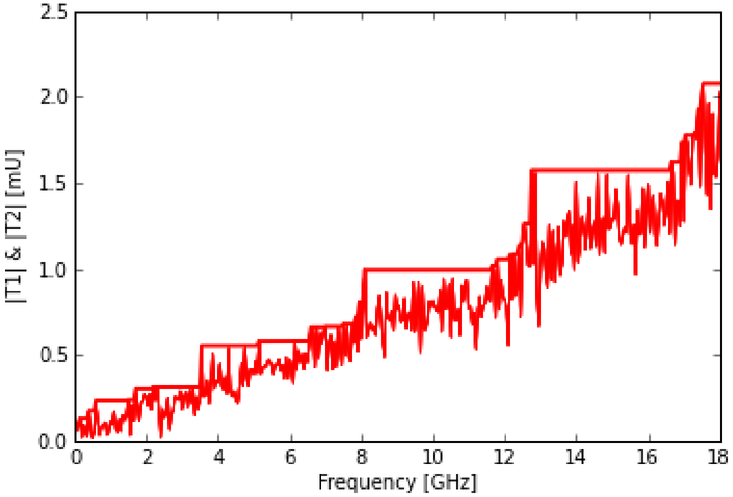
$S_{ii,\mathrm{line}}$ after SOLT calibration



$S_{ii,\mathrm{thru}} - S_{ii,\mathrm{line}}$ after SOLR calibration



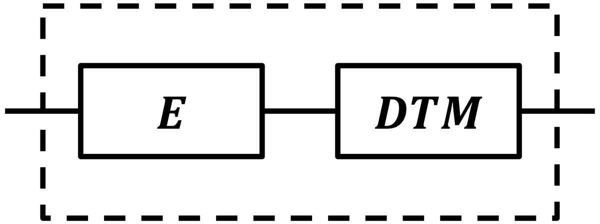
$S_{ij,\mathrm{thru}} - 1$ after SOLR calibration



Calibrating a one-port calibration kit

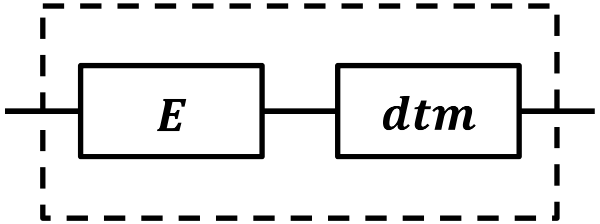
- Calibration comparison

Calibration using master standards



$\boldsymbol{e}$

Calibration using DUT



$\boldsymbol{e}'$

$$\boldsymbol{DTM} = \begin{pmatrix} \frac{D}{\sqrt{1+T}} & \sqrt{1+T} \\ & M \end{pmatrix}$$

$$\boldsymbol{\delta\tau\mu} = \boldsymbol{e}^{-1} \otimes \boldsymbol{e}' = \boldsymbol{DTM}^{-1} \otimes \boldsymbol{dtm}$$

$$\boldsymbol{dtm} = \begin{pmatrix} \frac{d}{\sqrt{1+t}} & \sqrt{1+t} \\ & m \end{pmatrix}$$

$$d \approx \delta(1+T) + D + M\delta^2$$

$$t \approx \tau + T + \tau T + 2M\delta$$

$$m \approx \mu + M(1+\tau)$$

$\Rightarrow$ After calibration we will have d, t, and m with values and uncertainties

$$\boldsymbol{\delta\tau\mu} = \begin{pmatrix} \frac{\delta}{\sqrt{1+\tau}} & \sqrt{1+\tau} \\ & \mu \end{pmatrix}$$

Comparison of two-port ripple technique and calibration comparison

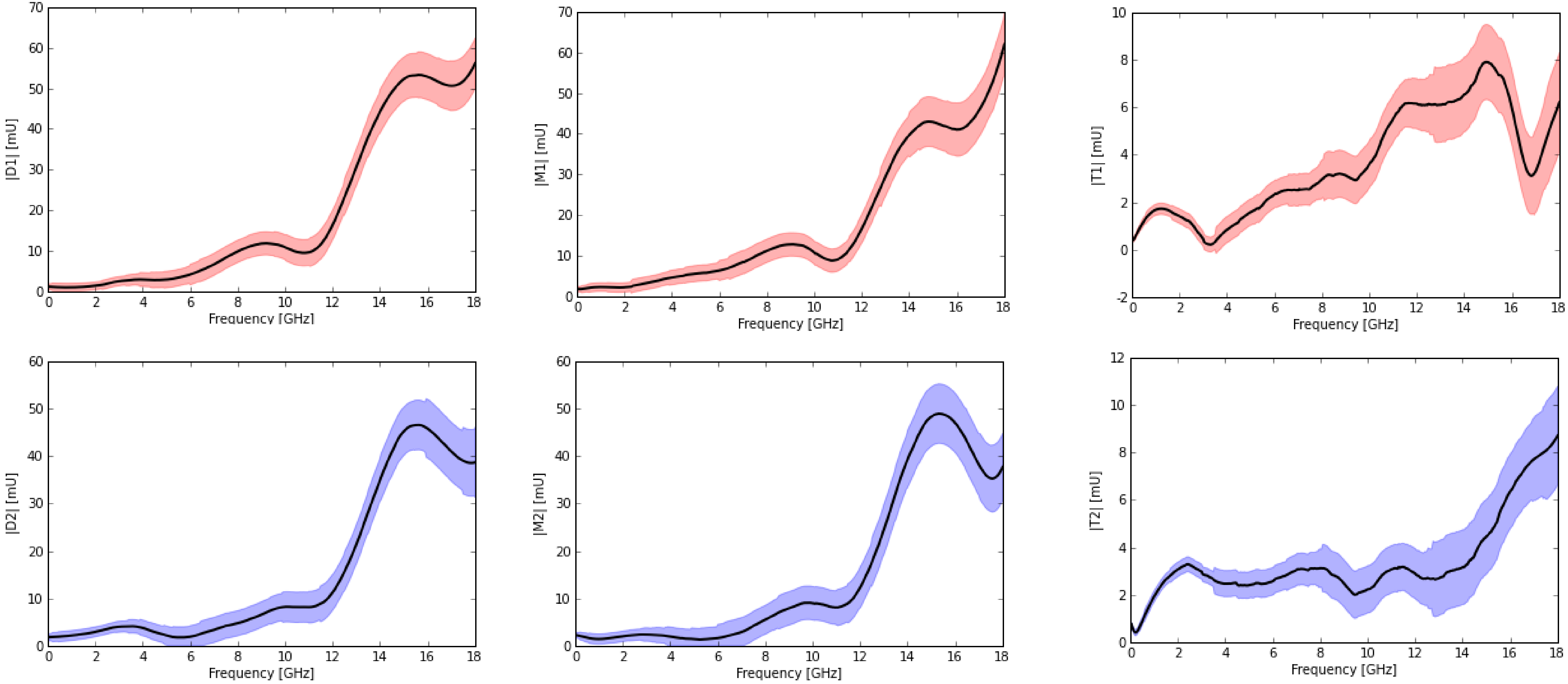
Master standard

1. One calibration using table based data for calibration standards (Type-N SOLT kit)
2. Use two-port ripple technique to obtain DTM (nominally zero)

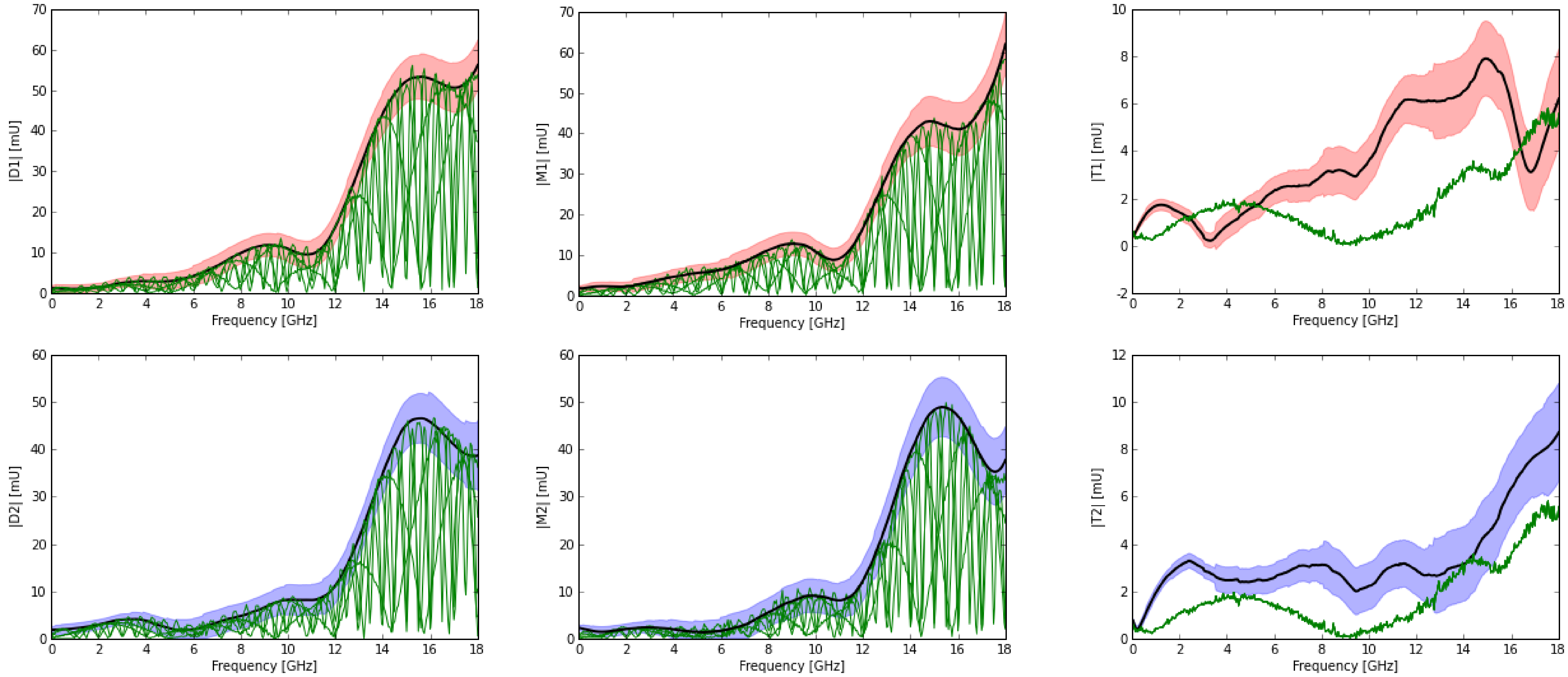
DUT

3. One calibration using manufacturer model
4. Use calibration comparison technique to obtain dtm
5. Use two-port ripple technique to obtain dtm

Comparison to two-port ripple technique



Comparison to two-port ripple technique



Conclusions

- Calibration kit uncertainty model (residual error model)
  - Equivalence to calibration standard model for SOL
  - Computation of uncertainties for measurements using the calibration kit uncertainty model
  - Assess uncertainty components using the two-port ripple technique
- Calibrating the ECU using another calibrated calibration kit
  - Keeping old tables for the calibration states
    - determine residual errors with uncertainties by calibration comparison
    - measurement uncertainty given by combination of residual error and uncertainty in residual error
    - second tier correction (i.e. post-correction using the residual errors ⇒ final uncertainties only from uncertainty in residual errors (Needs more work)
  - With updated tables for the calibration states
    - Residual errors will become nominally zero with uncertainty

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