Evaluation of relativistic effects in Two-Way Satellite Time and Frequency Transfer

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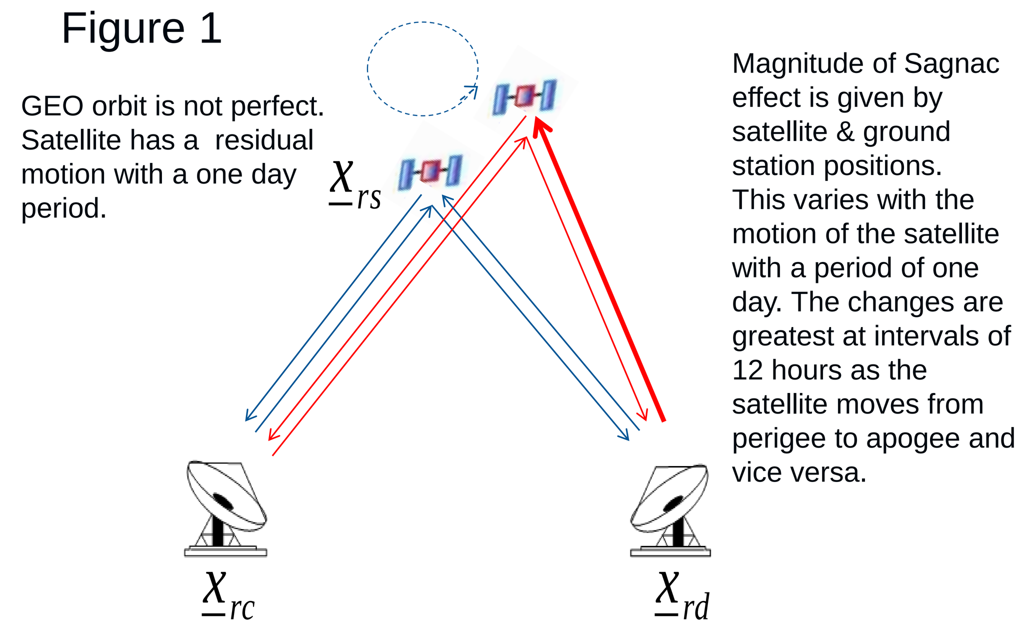
Introduction

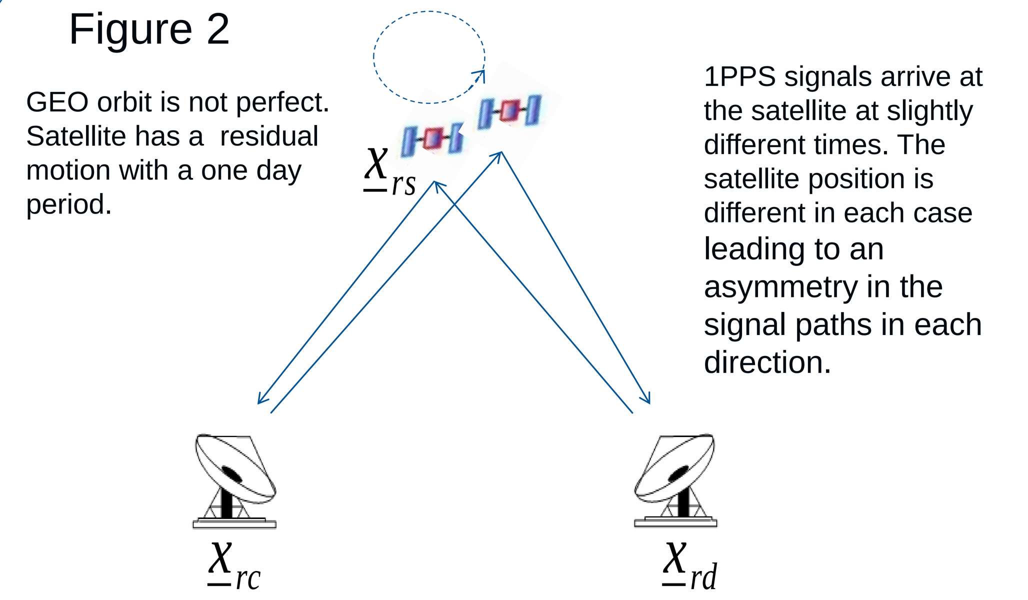
A method is described for applying existing theory to evaluate and correct relativistic effects in Two-Way Satellite Time and Frequency Transfer (TWSTFT). This work is part of the International Time scales with Optical Clocks (ITOC) European Metrology Research Programme project and also provides an input to a TWSTFT broadband experiment. The latter is to compare ground-based optical frequency standards between the four NMIs INRIM, NPL, OP and PTB. Although the aim is to achieve an experimental uncertainty of 10^{-16} in frequency transfer, relativistic corrections have been evaluated to 5 parts in 10^{17}. Results from the TWSTFT broadband experiment are reported in a related poster 'Satellite link performance for optical clock comparison'.

Figure 1



This can be calculated by knowing the position of the satellite and ground stations to sufficient accuracy.

Figure 2



A varying difference between two paths leads to a frequency transfer instability. Can reduce this by applying an offset, Δt, between the 1PPS transmit times of the two stations.

The equation below from Petit & Wolf 1994, gives the two corrections necessary to ensure TWSTFT time comparisons to an accuracy of 1 ps.

Correction due to Sagnac effect = δ_{Sagnac} Variation in signal path delays due to satellite motion = δ_{motion}

$$\delta_{corr} = \frac{\overrightarrow{R_{cd}}\cdot(\vec{\omega}\times\overrightarrow{x_{rs}})}{c^2} + \frac{(R_{cs}-R_{ds}-c\Delta t)\left(R_{ds}\overrightarrow{R_{cs}}+R_{cs}\overrightarrow{R_{ds}}\right)\cdot\overrightarrow{v_r}}{2R_{cs}R_{ds}c^2}$$

Here, δ_{corr} is the total relativistic correction, $\overrightarrow{R_{cd}}$ is the vector from the first ground station (labelled c) to the second ground station (labelled d), $\vec{\omega}$ is the vector of the rotational velocity of the Earth pointing along the axis of rotation, $\overrightarrow{x_{rs}}$, is the vector from the centre of the Earth to the satellite, , $\overrightarrow{R_{cs}}$ and $\overrightarrow{R_{ds}}$ are the vectors from the first and second ground stations to the satellite, Δt is the time-offset between the pulse transmit times of the two ground stations, $\overrightarrow{v_r}$ is the residual velocity vector of the satellite in the rotating frame of the Earth and c is the speed of light.

The first correction, referred to here as δ_{Sagnac}, is for the Sagnac effect described in Figure 1. The derivative of this term with respect to time gives the average fractional frequency shift correction. The second term, referred to as δ_{motion}, corrects for the different signal path lengths and is described in Figure 2. Instead of evaluating and correcting it in post-processing, it was decided to eliminate it to an appropriate uncertainty level by applying a deliberate `ideal time-offset', Δt_{ideal}, in the pulses transmitted by the ground stations such that their corresponding pulses arrive at the satellite sufficiently close in time. This time-offset was calculated based on the second term in the equation, where $\Delta t = (R_{cs} - R_{ds})/c$

Selecting a satellite: SES ASTRA3B

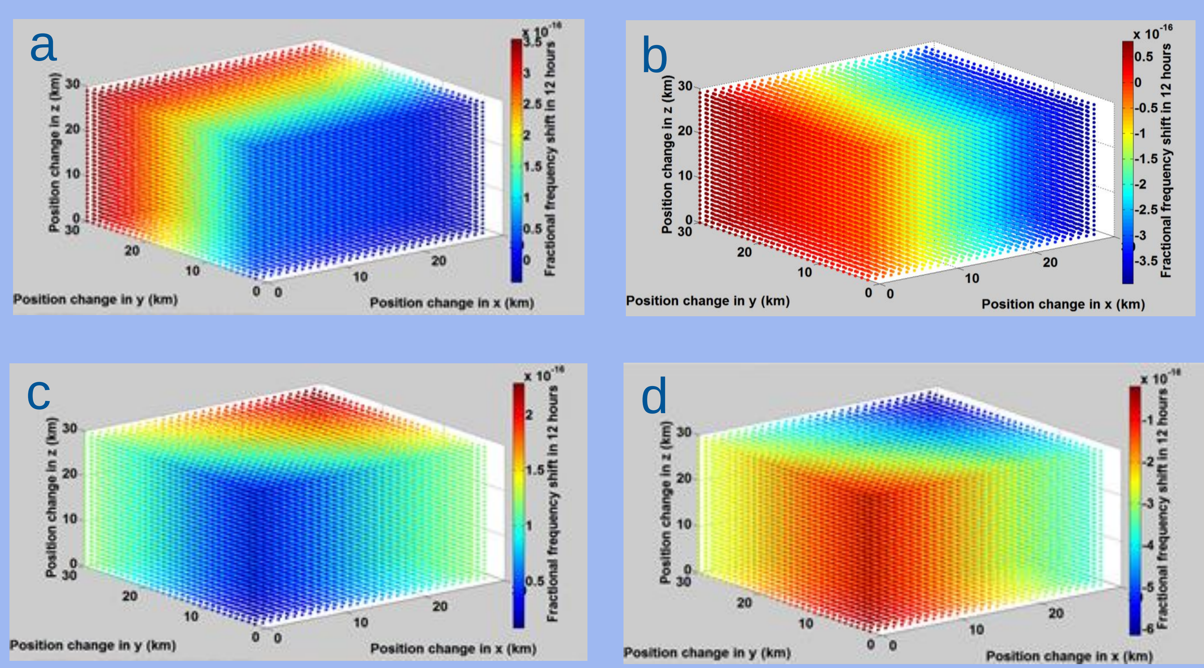


Figure 3: Simulations of sagnac average fractional frequency shift corrections for the links a) INRIM-PTB, b) OP-NPL, c) NPL-PTB and d) NPL-INRIM using Telstar 11N. The radial distance to the Telstar 11N satellite varies by approximately 30 km between perigee and apogee.

It was necessary to identify a satellite operator able to provide position data for a suitable satellite to sufficient accuracy to correct for the Sagnac effect. Consequently, an estimate of the position requirements first needed to be determined. Simulations were used to estimate the size of the Sagnac effect for Telstar 11N, used for routine TWSTFT between INRIM, NPL, OP and PTB. The plots in Figure 3 show the Sagnac average fractional frequency shift for four links using Telstar 11N. The origin of each plot represents the perigee of the satellite in its orbit. The variation in Sagnac shift over 12 hours is shown for all possible apogee locations of the satellite 12 hours later in a volume given by x, y and z co-ordinate changes of up to 30 km in the Earth-Centred Earth-Fixed (ECEF) system. The corrections in the radial and tangential directions could be up to 6 parts in 10^{16} at 12 hour intervals. It is found that a satellite position accuracy of 2 km or better is required to correct the Sagnac effect to an uncertainty of 5×10^{-17}. This agreed with a cross-check carried out by OBSPARIS. The satellite SES ASTRA3B was selected, as SES was able to provide adequate position data for this. Plots of the satellite position and velocity are shown in Figures 4 and 5. Satellite station-keeping manoeuvres occurred four times in the period.

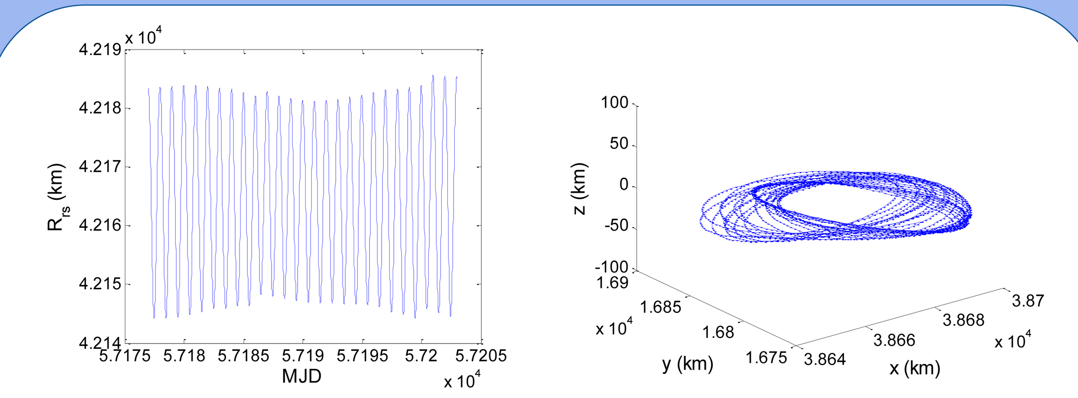


Figure 4. ASTRA3B position in ECEF system and distance of satellite from the centre of the Earth, R_{rs}.

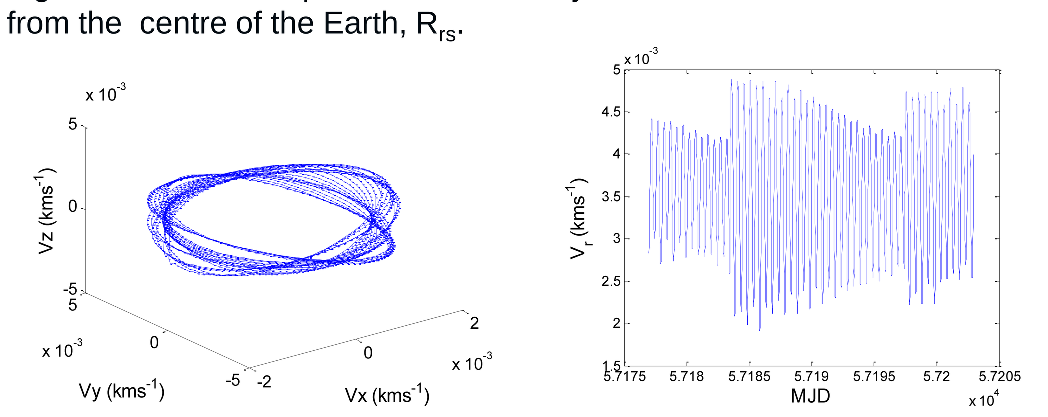


Figure 5: ASTRA3B velocity in ECEF system and its magntitude, V_r.

Evaluating effects for SES ASTRA3B

Relativistic effects were evaluated for a 26-day TWSTFT broadband experiment between the four NMIs INRIM, NPL, OP and PTB in June 2015. The plots below show the value of δ_{Sagnac} against MJD for the PTB-OP and PTB-NPL links. A time-offset was applied to the transmit times of the PTB and OP ground stations to reduce the impact of the variation in signal path-delays to lower than 5×10^{-17} in average fractional frequency shift over 12 hours. A time-offset was not necessary for the NPL ground station. No time-offset was applied to the INRIM ground station which meant that effects of both δ_{motion} and δ_{Sagnac} were present in the frequency transfer for all INRIM links.

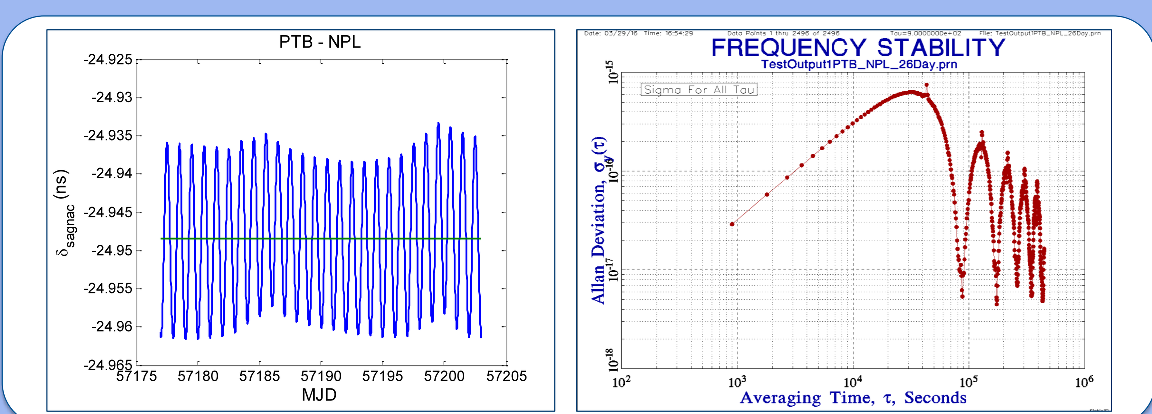


Figure 6: Plots of the δ_{Sagnac} against MJD and ADEV of δ_{Sagnac} for the PTB-NPL link.

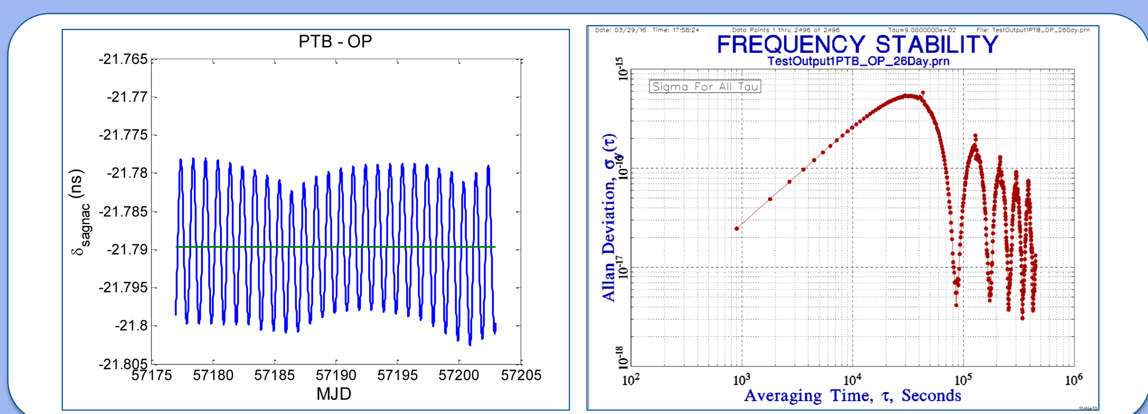


Figure 7: Plots of the δ_{Sagnac} against MJD and ADEV of δ_{Sagnac} for the PTB-OP link.

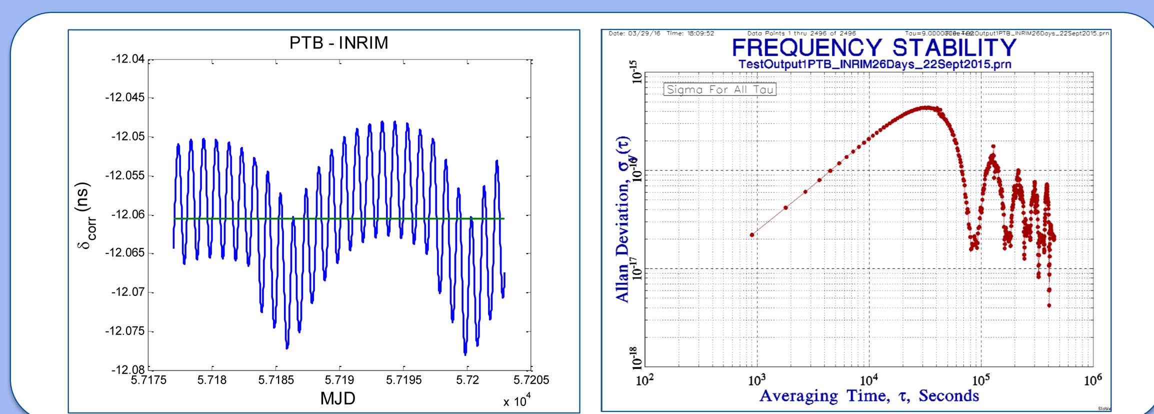


Figure 8: Plots of δ_{corr} $(= \delta_{Sagnac} + \delta_{motion})$ against MJD and ADEV of δ_{corr} for the PTB-NPL link.

The plots show that the ADEV of δ_{Sagnac} in the PTB-NPL and PTB-OP links and $\delta_{corr} = \delta_{Sagnac} + \delta_{motion}$ in the PTB-INRIM link at averaging times of 12 hours can be up to several parts in 10^{16}. However, the ADEV is much smaller at 1 day averaging times, equivalent to ~1 part in 10^{17}. Consequently, the above effects would contribute to the total uncertainty of the TWSTFT links, especially at averaging times of 12 hours, but they may not be apparent if other sources of uncertainty, such as due to temperature variations, are significantly higher.

Conclusions

The method described in Petit & Wolf 1994 has been used to evaluate, and in some cases substantially reduce, relativistic effects in TWSTFT. The impacts of the Sagnac effect and signal path asymmetry due to satellite motion have been addressed. These have been evaluated or eliminated to an uncertainty level of 5 parts in 10^{17} for a 26-day TWSTFT broadband experiment. It was found that the magnitudes of these effects can be up to a few parts in 10^{16} at averaging times of 12 hours and of order 10^{-17} at one day.

Acknowledgements

We acknowledge a consistency check carried out by Pacôme Delva of OBSPARIS on the satellite position accuracy requirements.

References: G. Petit & P. Wolf, "Relativistic theory for picosecond time transfer in the vicinity of the Earth", Astronomy and Astrophysics, 286, pp. 971-997 (1994)