Relativistic Corrections for Time and Frequency Transfer in Optical Fibres

Jan Geršl*, Pacôme Delva and Peter Wolf****

*Czech Metrology Institute, Okružní 31, 63800 Brno, Czech Republic

** LNE-SYRTE, Observatoire de Paris, CNRS UMR8630, UPMC, LNE, 61 avenue de l'Observatoire, F-75014 Paris, France

e-mail: **jgersl@cmi.cz**

European Metrology Research Programme
▸ Programme of EURAMET

The EMRP is jointly funded by the EMRP participating countries
within EURAMET and the European Union

Abstract

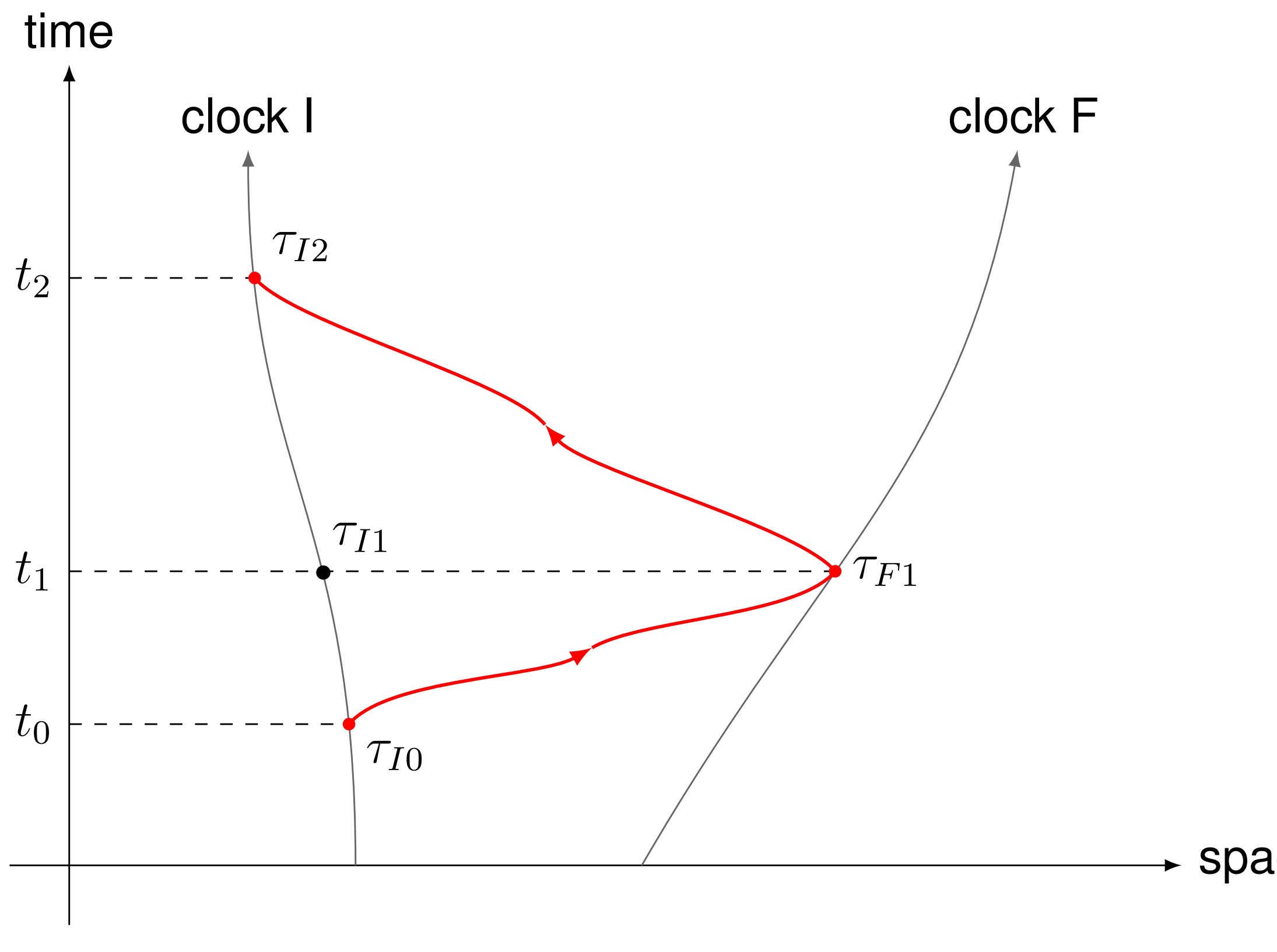
Atomic clocks have been improving rapidly over the past years and are now reaching stabilities and accuracies of a few parts in 10^{18} in fractional frequency. This pushes also the evaluation of phenomena that affect the signal propagation during the clocks comparisons to new levels of uncertainty. One ensemble of such phenomena are the relativistic effects for a signal propagation in optical fibre which is moving with given velocity due to the Earth surface motions (rotation, tides) and which is exposed to the Earth gravity field.

In this work within the ITOC project we derive relativistic corrections for one-way and two-way time and frequency transfer over optical fibres neglecting no terms that exceed 1 ps in time and 10^{-18} in fractional frequency, and estimate their magnitude in typical fibre links. We also provide estimates of the uncertainties in the evaluation of the relativistic corrections due to imperfect knowledge of parameters like the coordinates of the fibre and stations, Earth rotation, or thermal effects of the fibre index and length. The links between Teddington(UK) and Paris(F) as well as Braunschweig(D) and Paris(F), that are currently under construction, are studied as specific examples.

Details of this work can be found in J. Geršl, P. Delva and P. Wolf, Metrologia **52** (2015) 552

Time transfer

We consider a signal emitted from observer I at Geocentric Coordinate Time t_0 corresponding to proper time τ_{I0} of his clock I. The signal propagates in an optical fibre on Earth's surface and is received by observer F at coordinate time t_1 corresponding to proper time τ_{F1} of his clock F (see the figure below). Then a signal is sent from observer F at coordinate time t_1 and received by observer I at time t_2 corresponding to proper time τ_{I2} of clock I. Using the coordinate time synchronisation convention we define $\tau_{I1} = \tau_I(t_1)$.



The desynschronisation of the clocks $\tau_{F1} - \tau_{I1}$ can be determined using the proper times measured by the clocks and using the calculated propagation times $\Delta t_+ = t_1 - t_0$ and $\Delta t_- = t_2 - t_1$. We obtain the following formula containing relativistic corrections up to the 1 ps level

$$\Delta t_\pm = \frac{1}{c}\int_0^L n\,\mathrm{d}l \pm \frac{2\boldsymbol{\omega}\cdot\mathbf{A}}{c^2} \pm \frac{1}{c^2}\int_0^L \mathbf{v}_R\cdot\mathbf{s}_l\,\mathrm{d}l + \frac{1}{c^3}\int_0^L n\left(w + v^2/2\right)\mathrm{d}l$$

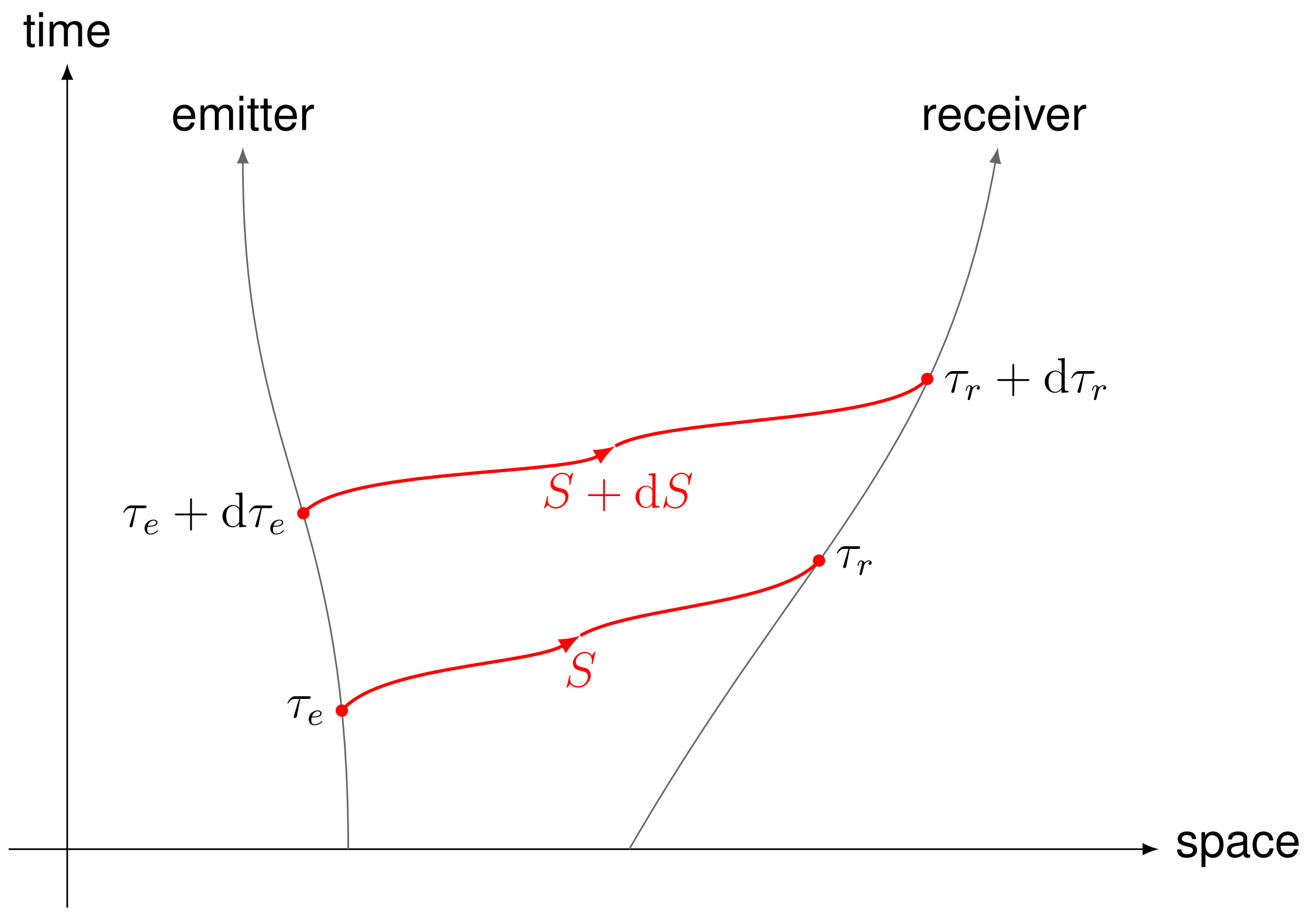
where we integrate along the rest length parameter of the fibre l at time t_1. The first term is the classical Newtonian term containing the effective refractive index n, the second term is the Sagnac correction where $\boldsymbol{\omega}$ is angular velocity vector of the Earth and $\mathbf{A}$ is the Sagnac area (vector) of the fibre, in the third term $\mathbf{v}_R$ is a velocity of the fibre in the frame co-rotating together with the Earth (usually very small) and $\mathbf{s}_l$ is tangent of the fibre with the parameter l, the last term is a part of the Shapiro correction where w is the Newtonian gravitational potential and $v^2/2$ is centrifugal potential due to the Earth rotation.

The table below summarises typical values of contributions to propagation time for 1000 km long fibre:

Effect	Contribution per 1000 km
Length and refractive index of the fibre (Newtonian term; 1-way only)	5 ms
Velocity of the fibre due to the Earth rotation (Sagnac)	5 ns
Velocity of the fibre due to the Earth tides	0.3 fs
Gravitational plus centrifugal potential on the Earth surface (1-way only)	3 ps

Frequency transfer

For frequency transfer the theoretical frequency shift of the signal propagating in optical fiber needs to be known. We consider two clocks measuring proper time along their trajectory. One signal with phase S is emitted at proper time τ_e, and another one with phase $S + \mathrm{d}S$ at time $\tau_e + \mathrm{d}\tau_e$. They are received respectively at time τ_r and $\tau_r + \mathrm{d}\tau_r$ (see the figure below). The proper frequency measured by the emitter/receiver is respectively: $\nu_{e/r} = \frac{1}{2\pi}\frac{\mathrm{d}S}{\mathrm{d}\tau_{e/r}}$



For the ratio of the received and emitted frequency the following formula containing relativistic corrections up to the 10^{-18} level was derived

$$\frac{\nu_r}{\nu_e} = 1 + \frac{1}{c^2}(w_r - w_e) - \frac{1}{c}\int_0^L \left(\frac{\partial n}{\partial t} + n\alpha\frac{\partial T}{\partial t}\right)\mathrm{d}l - \frac{1}{c^2}\int_0^L \mathbf{a}\cdot\mathbf{s}_l\,\mathrm{d}l$$

where the first correction is the well known gravitational red shift with $w_{e/r}$ being the gravitational potential at the events of emission or reception, the second correction is a Doppler shift due to changes in the optical length of the fibre with T being temperature of the fibre and α its thermal expansion coefficient and the third correction is due to the fibre's acceleration in the GCRS frame including the centrifugal, Coriolis and Euler acceleration due to the Earth rotation.

The table below summarises typical values of contributions to relative frequency shift for 1000 km long fibre:

Effect	Correction
Difference of gravitational plus centrifugal potential at endpoints	$> 10^{-18}$
Variations in length and refractive index due to temperature changes (1-way only)	$\sim 10^{-13}$
Coriolis acceleration of the fibre due to velocity of the Earth tides	8×10^{-20}
Euler acceleration of the fibre due to angular acceleration of the Earth rotation	4×10^{-20}
Acceleration of the Earth tides	3×10^{-20}

Uncertainties of input parameters

Uncertainties of parameters entering the formulas for time and frequency transfer have been studied and their values corresponding to 1 ps uncertainty in propagation time or 10^{-18} uncertainty in relative frequency shift have been determined. The illustrative values in the tables below were obtained for situations where the sensitivity of a correction to a parameter is maximized and they are calculated for 1000 km long fibre.

Example of input parameters and their maximal uncertainties sufficient for 1 ps uncertainty in time transfer:

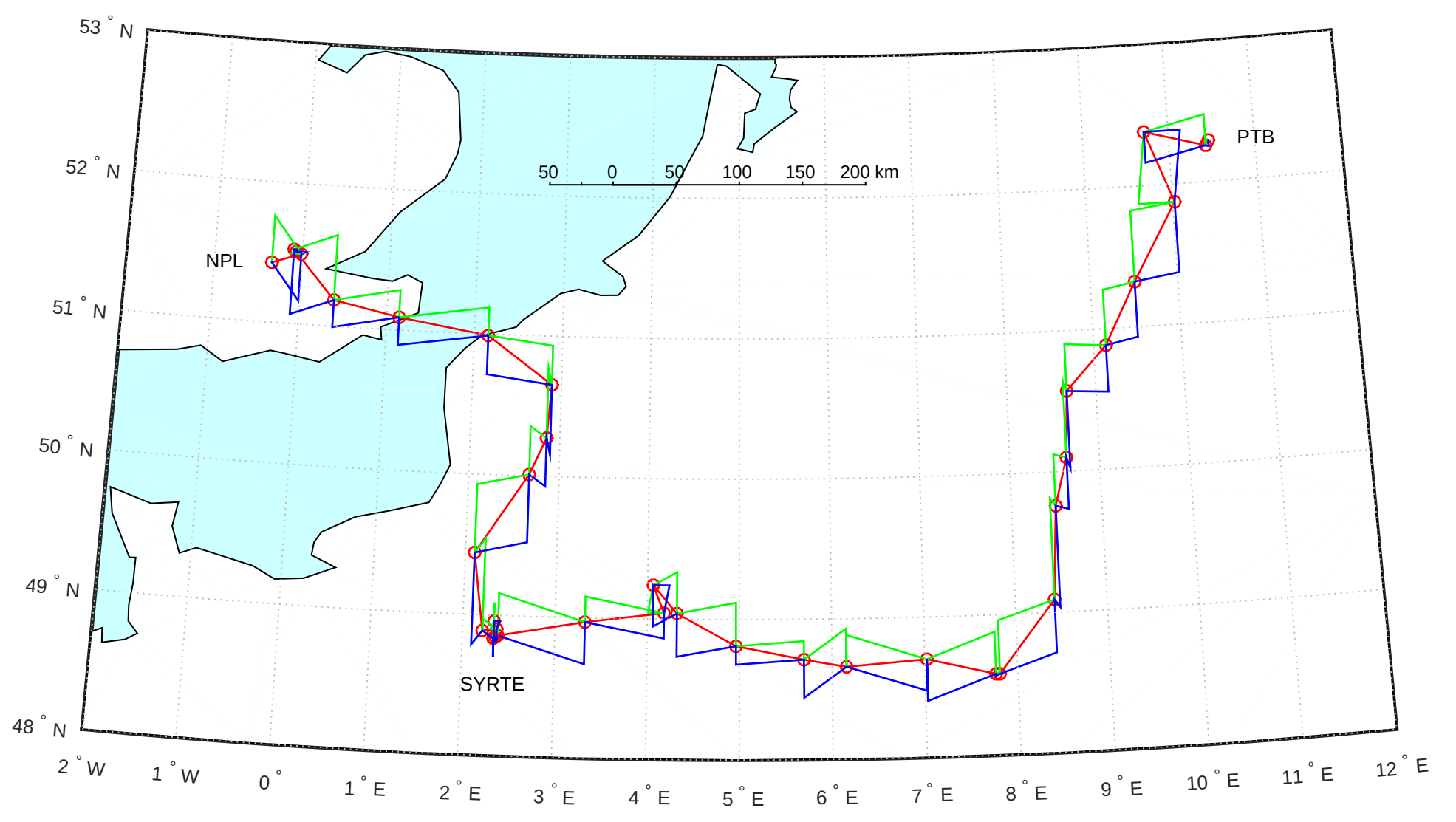
Parameter	Uncertainty
Fibre length (1-way only)	0.2 mm
Refractive index (1-way only)	3×10^{-10}
Fibre endpoints position (Sagnac)	200 m
Fibre inner points position (Sagnac)	600 m
Fibre velocity in co-rotating frame	9 cm/s
Earth angular velocity	$\sim$ 0.01 % (relative)
Gravitational plus centrifugal potential (1-way only)	$\sim$ 30 % (relative)

Example of input parameters and their maximal uncertainties sufficient for 10^{-18} uncertainty in relative frequency shift (parameters related to the gravitational redshift are not discussed here):

Parameter	Uncertainty
Time derivative of the fibre temperature (change of length and refractive index; 1-way only)	3×10^{-11} K/s
Fibre velocity in co-rotating frame	0.6 mm/s
Fibre acceleration in co-rotating frame	9×10^{-8} ms^{-2}
Fibre position	$>$ Earth radius
Earth angular velocity	$>$ 100 % (relative)
Earth angular acceleration	$>$ 100 % (relative)

Application to developed fibre links

Fibre links between NPL (Teddington) and SYRTE (Paris) and between SYRTE (Paris) and PTB (Brauschweig) are currently under construction. The results of this work were used to calculate the relativistic corrections for these links, especially the Sagnac correction for time transfer. Known positions of the shelters along the fibres and fibre length between the shelters were used. Exact fibre routing between the shelters was not known leading to an uncertainty in the Sagnac correction value:



fibre link	Length/km	Correction/ps
PTB-SYRTE	1401	3976 ± 27
NPL-SYRTE	813	1214 ± 6

Acknowledgements

The research has received funding from the European Metrology Research Programme (EMRP). The EMRP is jointly funded by the EMRP participating countries within Euramet and the European Union. The work have been done within an EMRP project International Time Scales with Optical Clocks and related Research Mobility Grant.