Least-squares analysis of clock comparison data to deduce optimized frequency and frequency ratio values

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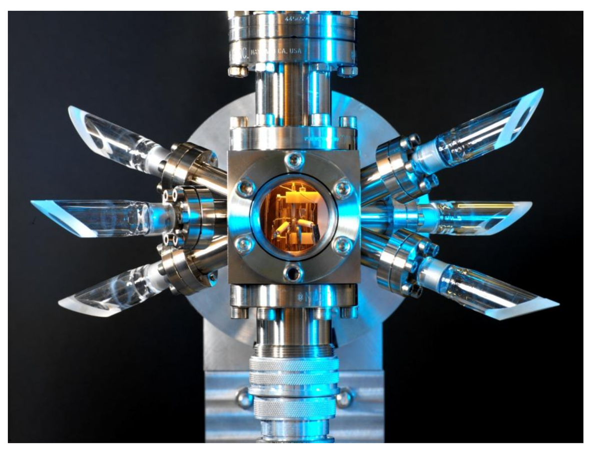
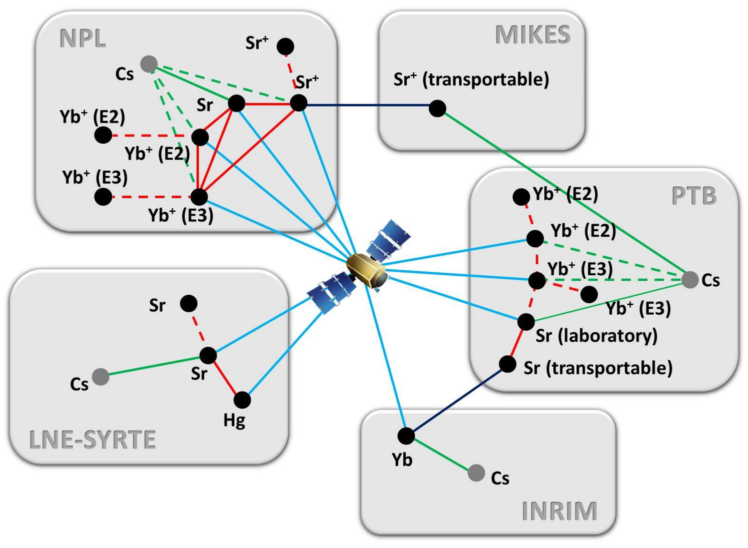
Optical clocks and the SI second



- Optical clocks have now reached levels of stability and accuracy that surpass the performance of the best caesium fountain primary standards
- Raises the prospect of a future **redefinition of the SI second**
- Optical clocks can already be used as **secondary representations of the second**
- CIPM recommended frequencies and uncertainties [1] are assigned by the CCL-CCTF Frequency Standards Working Group (WGFS)
- Almost all data considered so far comes from **absolute frequency measurements** of optical clocks relative to caesium primary standards
- However future information about the reproducibility of optical clocks will come mainly from direct **optical frequency ratio measurements**

Optical frequency ratio measurements

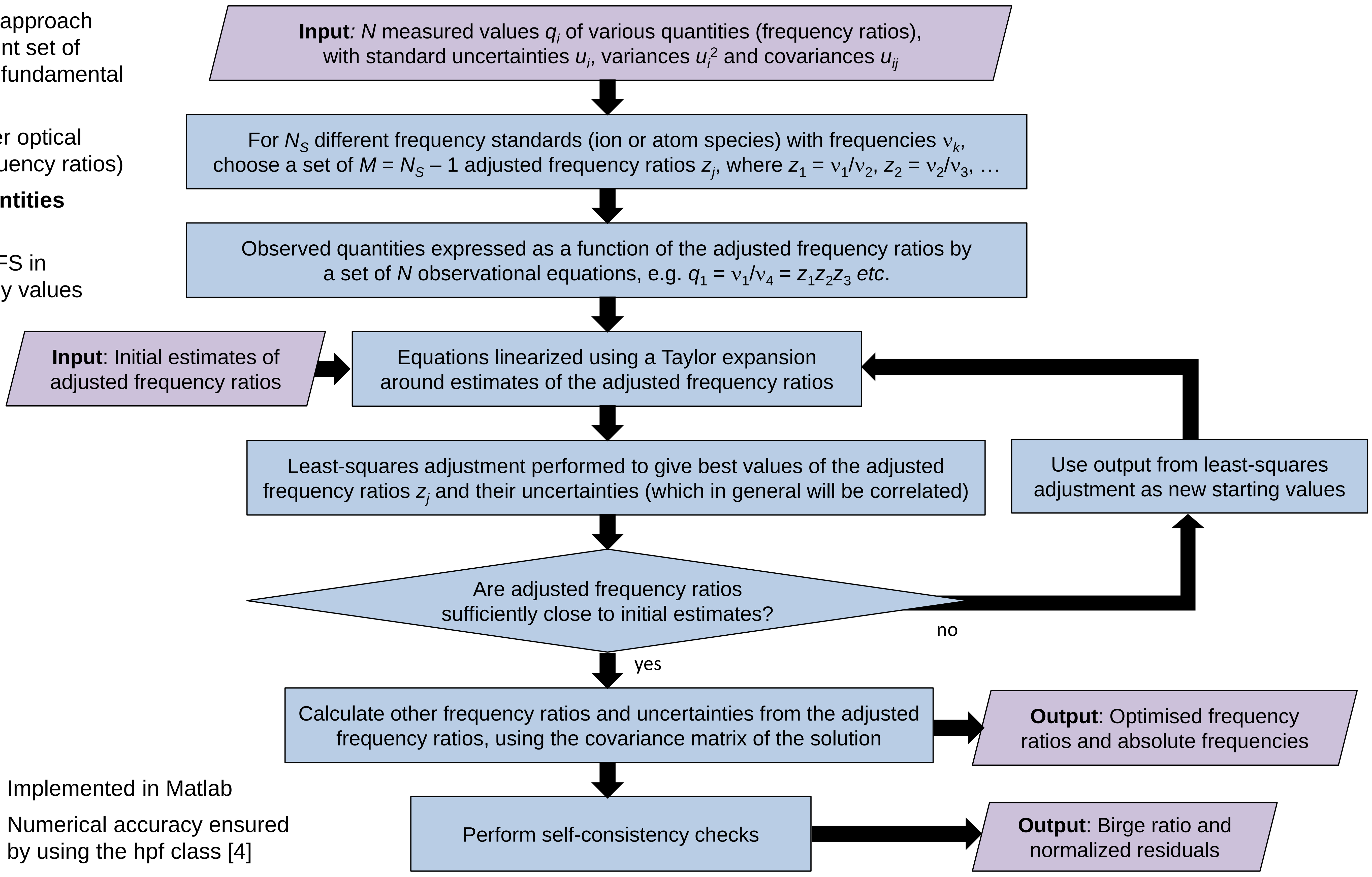
- Within the ITOC project [2], a coordinated programme of clock comparisons will lead to
  1) a set of frequency ratio measurements between all high accuracy optical clocks being developed in European NMIs
  2) a comprehensive set of absolute frequency measurements with uncertainties at the limit set by caesium primary standards



- This complete set of measurements will be **over-determined**, in the sense that it will be possible to deduce some of the frequency ratios from several different measurements
- New methods are required for analysing such over-determined sets of clock comparison data

Analysis procedure

- **Least-squares adjustment**, based on the approach used by CODATA to provide a self-consistent set of internationally recommended values of the fundamental physical constants [3]
- **All data stored as frequency ratios** (either optical frequency ratios, or optical-microwave frequency ratios)
- **Correlations between the measured quantities** included in the analysis
- Tested using input data considered by WGFS in deriving the CIPM recommended frequency values

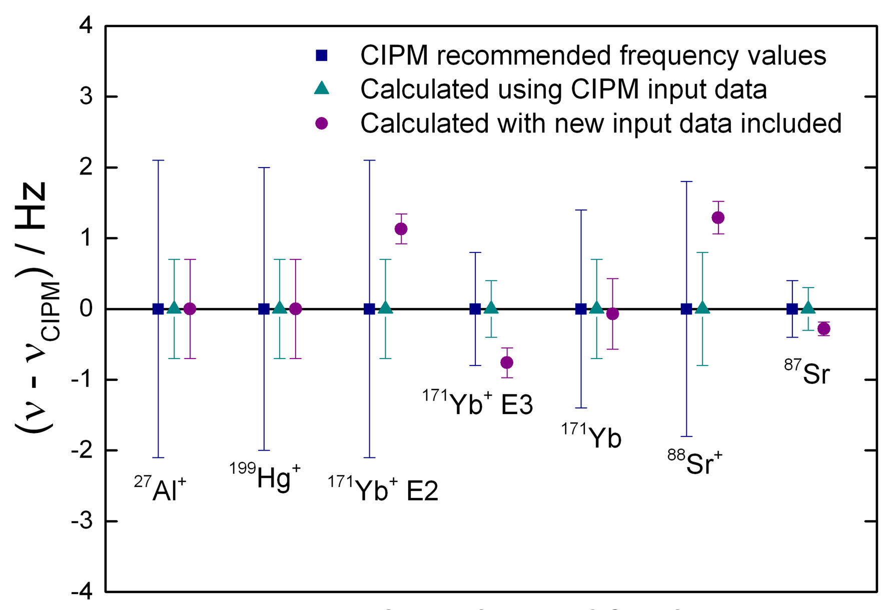


Transitions included in analysis:	
Atom/ion	**Transition**
^{1}H	1S – 2S
^{199}Hg	6s^{2} $^{1}S_0$ – 6s6p $^{3}P_0$
^{27}Al^{+}	3s^{2} $^{1}S_0$ – 3s3p $^{3}P_0$
^{119}Hg^{+}	5d^{10}6s $^{2}S_{1/2}$ – 5d^{9}6s^{2} $^{2}D_{5/2}$
^{171}Yb^{+}	6s $^{2}S_{1/2}$ – 5d $^{2}D_{3/2}$
^{171}Yb^{+}	6s $^{2}S_{1/2}$ – 4f^{13} 6s^{2} $^{2}F_{7/2}$
^{171}Yb	6s^{2} $^{1}S_0$ – 6s6p $^{3}P_0$
^{40}Ca	4s^{2} $^{1}S_0$ – 4s4p $^{3}P_1$
^{88}Sr^{+}	5s $^{2}S_{1/2}$ – 4d $^{2}D_{5/2}$
^{88}Sr	5s^{2} $^{1}S_0$ – 5s5p $^{3}P_0$
^{87}Sr	5s^{2} $^{1}S_0$ – 5s5p $^{3}P_0$
^{40}Ca^{+}	4s $^{2}S_{1/2}$ – 3d $^{2}D_{5/2}$
^{87}Rb	5s $^{2}S_{1/2}$ (F=1) – 5s $^{2}S_{1/2}$ (F=2)
^{133}Cs	6s $^{2}S_{1/2}$ (F=3) – 6s $^{2}S_{1/2}$ (F=4)

- Implemented in Matlab
- Numerical accuracy ensured by using the hpf class [4]

Results obtained from existing data

- Analysis software reproduces CIPM recommended frequency values
  - Exception is for ^{40}Ca (WGFS used an unweighted mean in this case)
- Uncertainties determined using analysis software are smaller than the uncertainties of the CIPM values
  - WGFS takes a conservative approach to estimating uncertainties because values are typically derived from only a few independent measurements



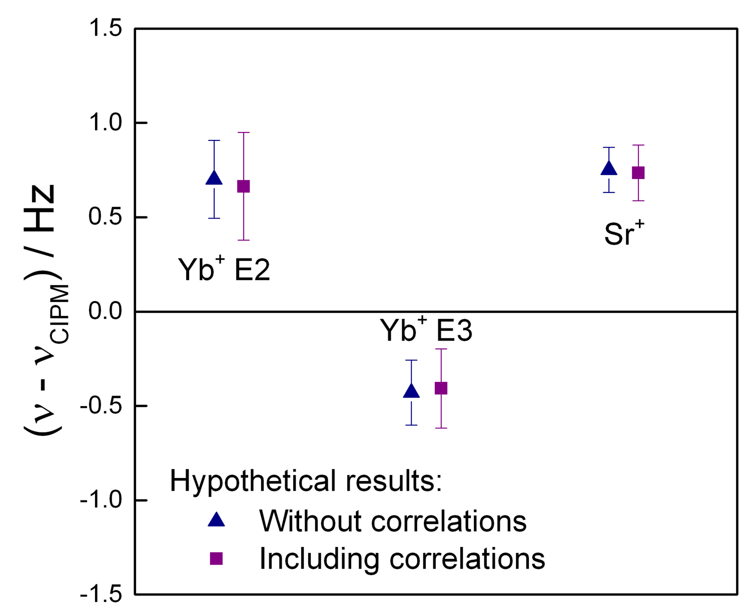
Frequency values obtained for the seven optical secondary representations of the second

- Inclusion of new data in analysis (recently published values + new measurements performed within the ITOC consortium) show the wisdom of this approach
- Above analysis neglects correlations between the input data

Importance of correlations

- Consider the hypothetical 10-day measurement campaign illustrated
- Each optical clock runs 60% of the time
- For the 6 measured frequency ratios, there are 12 non-zero correlation coefficients
- Correlations arise from both statistical and systematic uncertainties
- Correlation coefficients estimated from present stabilities and systematic uncertainties of NPL clocks

	1	2	3	4	5	6	7	8	9	10
Operational standards during 10-day measurement campaign										
Cs	■	■	■	■	■	■	■	■	■	■
Yb⁺ E2	■	■	■	■				■	■	
Yb⁺ E3			■	■	■			■	■	■
Sr⁺	■	■			■	■	■	■		
Measurements made during campaign										
Yb⁺ E2 / Cs	■	■	■	■				■	■	
Yb⁺ E3 / Cs			■	■	■			■	■	■
Sr⁺ / Cs	■	■			■	■	■	■		
Yb⁺ E2 / Yb⁺ E3			■	■				■	■	
Yb⁺ E2 / Sr⁺	■	■						■		
Yb⁺ E3 / Sr⁺					■			■		



- For specific values of the measured frequency ratios, the effect on the optimized frequency ratios and absolute frequencies can be determined
- Neglecting correlations leads to too much weight being given to these measurements
- This results in biased frequency values and underestimated uncertainties

Conclusion: More information is required about the correlations between the input data (for both intra-laboratory and inter-laboratory comparisons)

References

[1] http://www.bipm.org/en/si/si_brochure/appendix2/mep.html
[2] H, S. Margolis *et al*., pp. 908 – 911 in Proceedings of the Joint EFTF/IFCS (2013)
[3] P. J. Mohr and B. N. Taylor, Rev. Mod. Phys. 72, 351 (2000)
[4] Written by John R. D’Errico; available from the Matlab central file exchange

EMRP European Metrology Research Programme ▸ Programme of EURAMET
The EMRP is jointly funded by the EMRP participating countries within EURAMET and the European Union

This work was funded by the European Metrology Research Programme as part of the project “International timescales with optical clocks” (ITOC).
The EMRP is jointly funded by the EMRP participating countries within EURAMET and the European Union.

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