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Analysis of uncertainty in MRT dosimetry

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MetroMRT
Metrology for Molecular Radiotherapy

Outline

- Dosimetry chain
- Uncertainty analysis of links in chain
- Absorbed dose estimate and associated uncertainty
- Impact and conclusions

Uncertainty contributions

- Uncertainty contributions from all links in dosimetry chain
- Very many uncertainty sources
- Indicate major sources and their quantification
- Ultimate aim: realistic value for absorbed dose in volume of interest (VOI) and its associated uncertainty

Dosimetry chain

Measurement of administered activity

Definition and delineation of VOI

Activity determination in VOI by quantitative imaging

Time sequence of activity values

Integration procedure to determine cumulated activity

Absorbed dose calculation

General

- Model each link in the chain: mathematical/computational
- Apply recognized international uncertainty guidance
  - Best estimates of quantities involved
  - Associated standard uncertainties
- Repeated use of law of propagation of uncertainty

Absorbed dose calculation

$$D = \tilde{A}S$$

D absorbed dose in VOI

$\tilde{A}$ cumulated activity

S dose factor (MIRD S-factor)

$\tilde{A}$ given by area under the time-activity curve (TAC)

S given by MIRD tables or anthropomorphic phantom studies

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Absorbed dose uncertainty

$$D = \tilde{A}S$$

Relative standard uncertainties combine in quadrature [GUM]

$$\tilde{u}^2(D) = \tilde{u}^2(\tilde{A}) + \tilde{u}^2(S),$$

$$\tilde{u}(x) = \frac{u(x)}{|x|}$$

Relative standard uncertainties combine in quadrature [GUM]

Correlated $\tilde{A}$ and S:

$$\tilde{u}^2(D) = \tilde{u}^2(\tilde{A}) + \tilde{u}^2(S) + 2r\tilde{u}(\tilde{A})\tilde{u}(S),$$

r correlation coefficient between $\tilde{A}$ and S $(-1 \leq r \leq 1)$

Value used for r can make a big difference

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Need to evaluate r, $\tilde{u}(\tilde{A})$, $\tilde{u}(S)$

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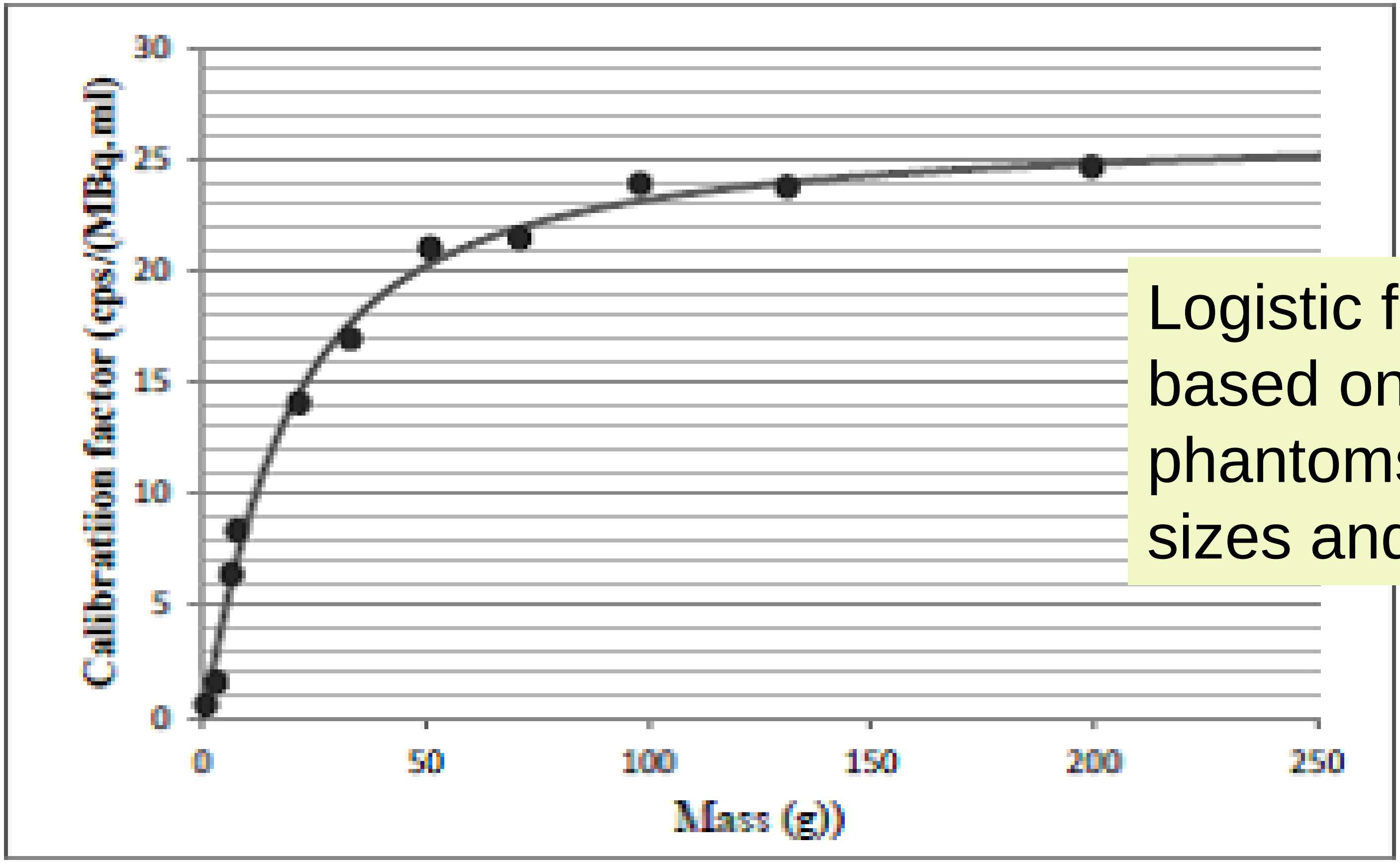
Construct time-activity curve (TAC)

- TAC based on some 3 – 6 time-activity points
- Each activity value deduced from scanned image at given time
- Activity values A_i have associated uncertainties
- The A_i correlated because of common effects
- $A_i = Q\bar{C}_i$
  - Q calibration factor specific to system/investigation
  - $\bar{C}_i$ mean counts in VOI at time t_i

Calibration factor $Q = Q(m)$

Calibration curve $Q(m)$ in terms of mass m in VOI

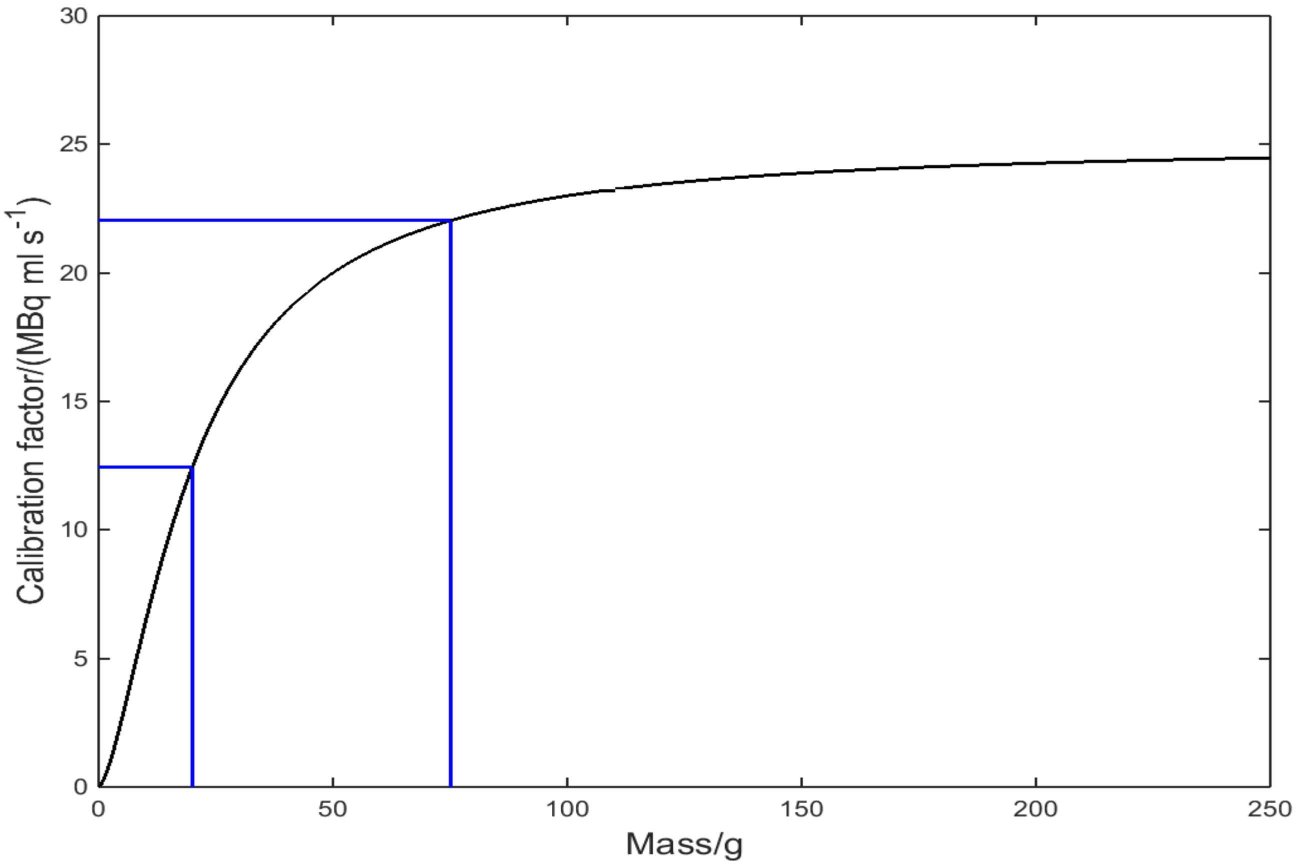


Logistic function based on scanning phantoms of different sizes and geometries

[Figure: ICR]

Calibration factor $Q = Q(m)$

Mass values → calibration factor values



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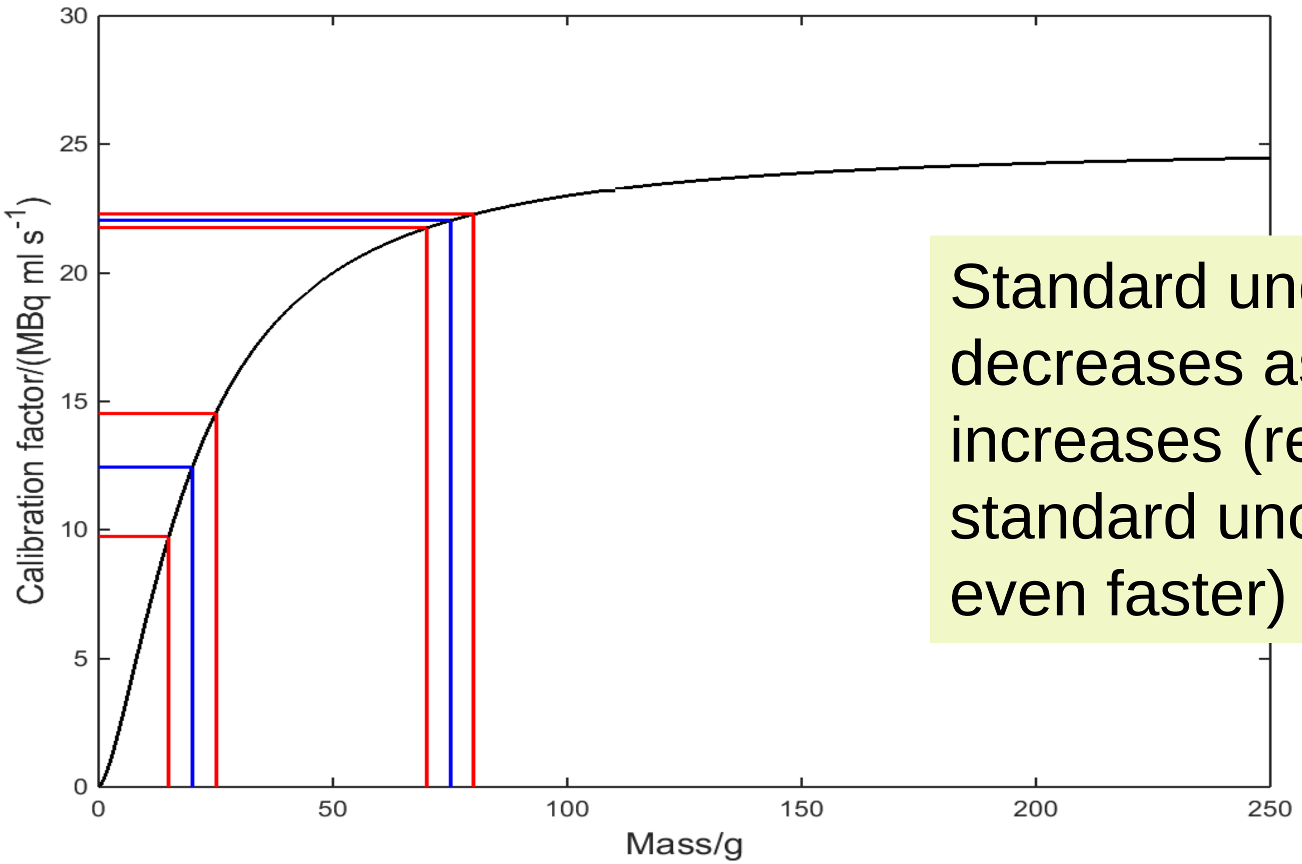
Calibration factor $Q = Q(m)$

Mass uncertainties → calibration factor uncertainties



Standard uncertainty decreases as mass increases (relative standard uncertainty even faster)

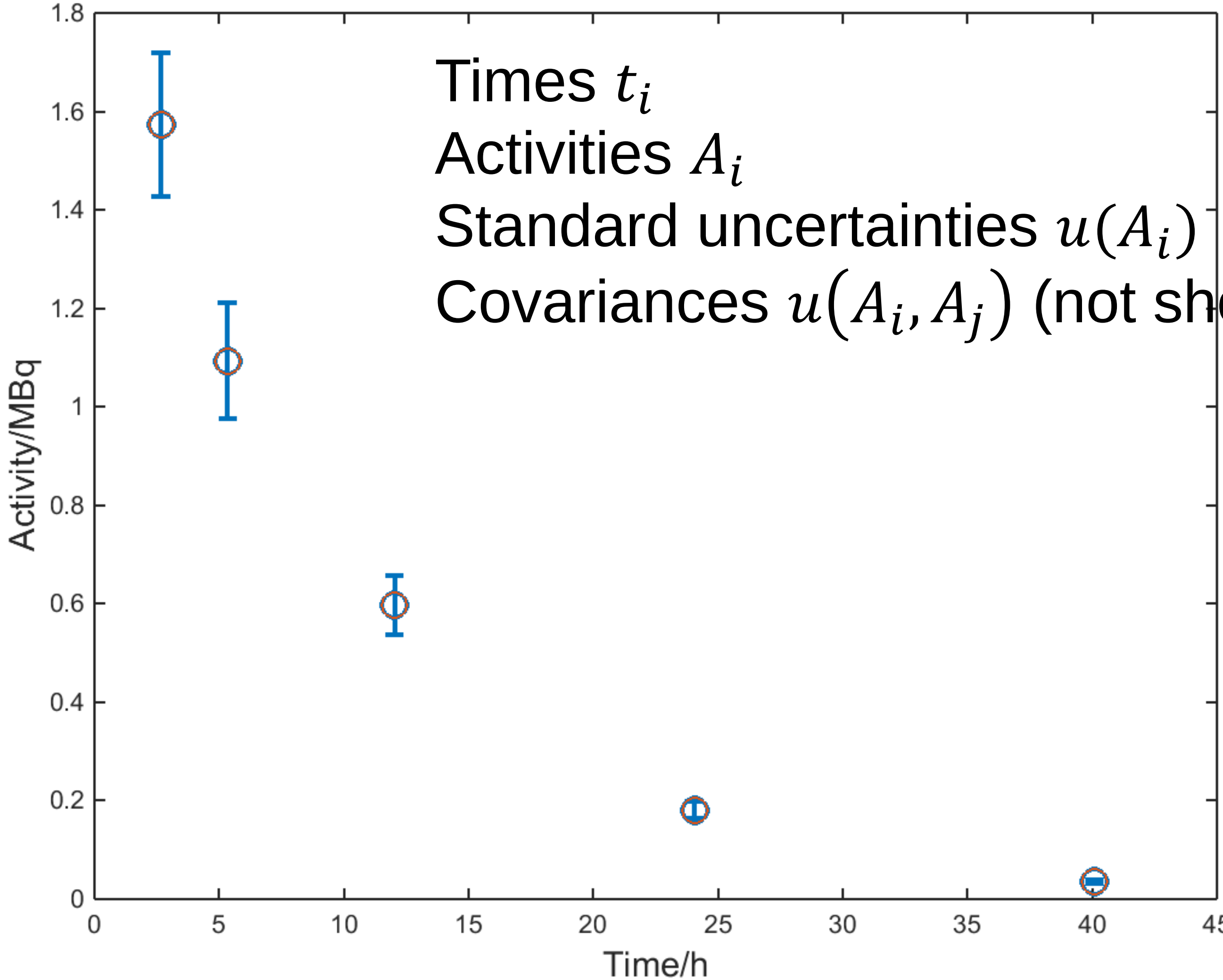
Time-activity data

Times t_i

Activities A_i

Standard uncertainties $u(A_i)$

Covariances $u(A_i, A_j)$ (not shown)



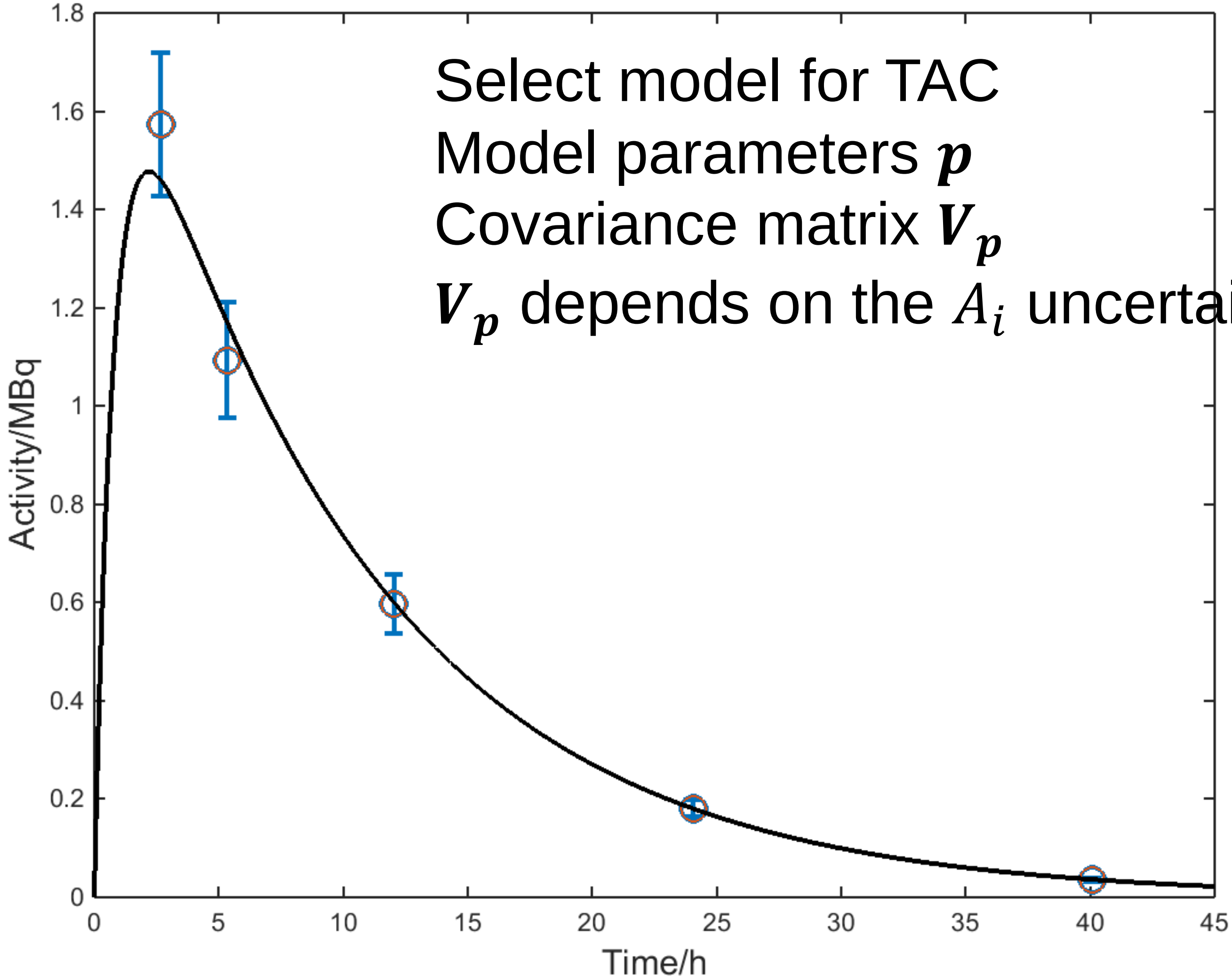
Time-activity curve

Select model for TAC

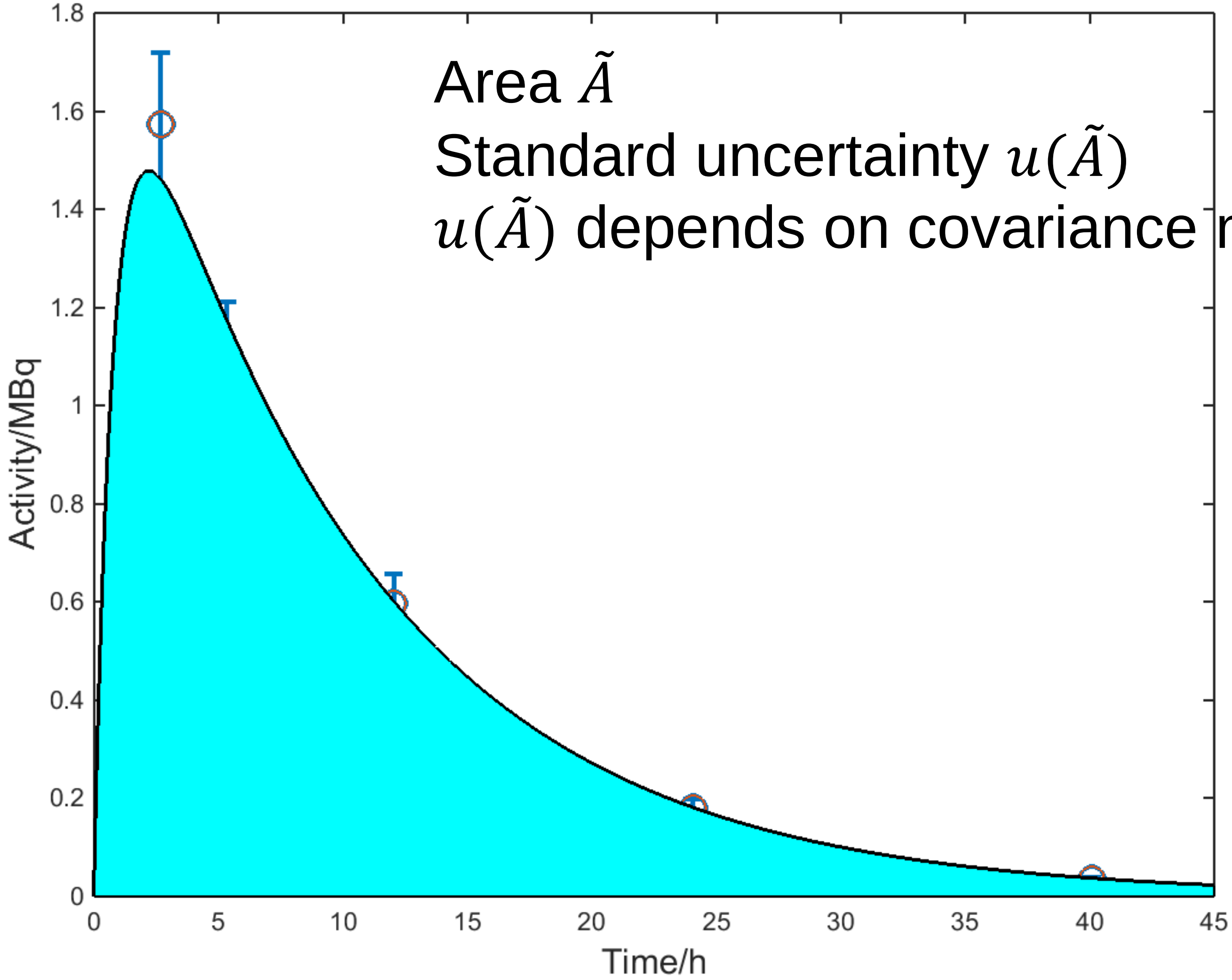
Model parameters $\boldsymbol{p}$

Covariance matrix $\boldsymbol{V_p}$

$\boldsymbol{V_p}$ depends on the A_i uncertainties



Cumulated activity



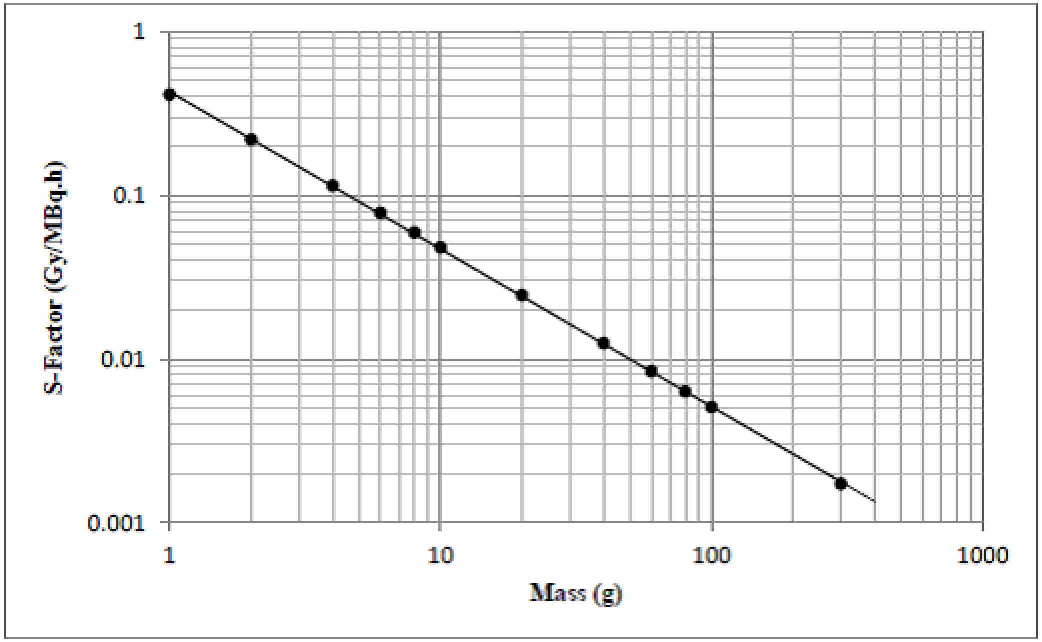
Area $\tilde{A}$

Standard uncertainty $u(\tilde{A})$

$u(\tilde{A})$ depends on covariance matrix $\boldsymbol{V_p}$

Dose factor S and its uncertainty

Make empirical fit to S-factor data against mass m in VOI (e.g., Buckley et al, 2007: unit density spheres of various masses)



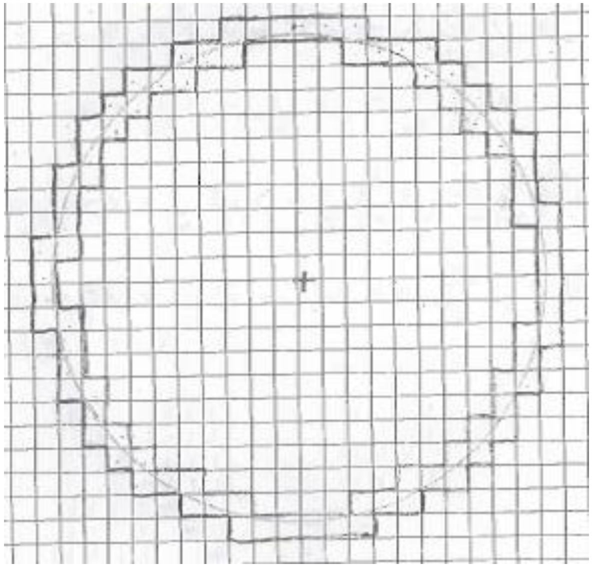
$$S = c_1 m^{-c_2}$$

[Figure: ICR]

$u(S)$ depends on knowledge of $u(m)$

$u(m)$ given by knowledge of VOI delineation

Definition and delineation of VOI



Estimate mean diameter of voxellized VOI
Diametral uncertainty limited by voxel size
Standard uncertainty associated with measured diameter d:

$$u(d) \approx 1 \text{ voxel side}$$

Use $m = Cd^3$ to obtain $\tilde{u}(m) = 3\,\tilde{u}(d)$

Relative uncertainty decreases as d increases

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Return to absorbed dose formula

$$D = \tilde{A}S$$

Both $\tilde{A}$ and S depend on mass m in VOI; thus correlated

Must use "complete" formula

$$\tilde{u}^2(D) = \tilde{u}^2(\tilde{A}) + \tilde{u}^2(S) + 2r\tilde{u}(\tilde{A})\tilde{u}(S)$$

Correlation coefficient r given by analysis of manner in which $\tilde{A}$ and S depend on mass m

Impact of uncertainty analysis and conclusions

Adoption of recognized uncertainty evaluation guidance → greater acceptance

Optimization of time points → reduction in number of such points for same absorbed dose uncertainty (respects clinical constraints relating to night-time working, etc.)

Uncertainty budget enables investigation of relative importance of links in metrology chain

Facilitates comparison of merits of dosimetry methodologies

Gives more complete information to decision makers