Sensitivity Study of the NIST 500 mm Guarded-Hot-Plate Apparatus: A Methodology Based on Orthogonal Designs

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EMRP THERMO: Metrology for Thermal Protection Materials
G15 CS4, National Physical Laboratory, Teddington UK (via teleconference)

Organized by:
National Physical Laboratory

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National Institute of Standards and Technology
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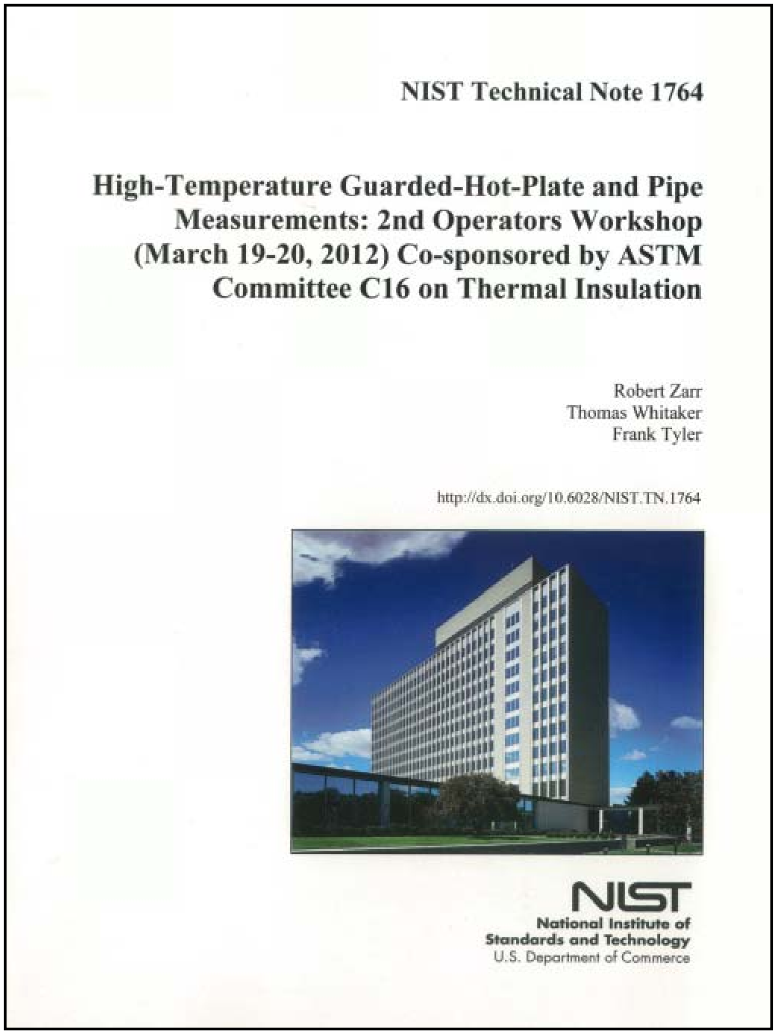
Motivation

2012 ASTM C16 Workshop



NIST Technical Note 1764

High-Temperature Guarded-Hot-Plate and Pipe Measurements: 2nd Operators Workshop (March 19-20, 2012) Co-sponsored by ASTM Committee C16 on Thermal Insulation

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Workshop Recommendation #2

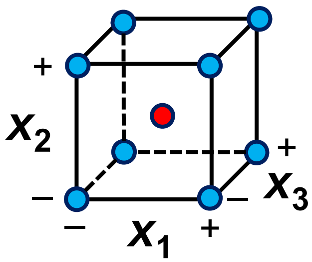
2) Attendees should consider conducting "in-house" sensitivity studies (also known by ASTM as "ruggedness tests")

Introduction to Design of Experiment (DEX)

James Filliben, NIST Mathematical Statistician

- **Orthogonal Factorial Design – Full (2^k) or Fractional (2^{k-p})**
  - Advantages:
    - Optimum design for examining multiple factors (screening process)
    - Balanced – every factor setting occurs the same number of times
    - Randomized – minimizes bias (i.e., drift protection over time)
    - Allows detection of interaction effects (not possible with one-factor-at-a-time design)
  - Disadvantage: $n = 2^k$; n increases geometrically with k (time ↑, cost ↑)
  - Disadvantage: $n = 2^{k-p}$; confounding (cannot estimate all effects separately)

- ***Example:***
  - Full factorial hypothetical small experiment
    - 3 factors ($k = 3$): $\boldsymbol{x_1}$, $\boldsymbol{x_2}$, $\boldsymbol{x_3}$
    - 2 settings: coded as –1 and +1
  - Number of runs, $n = 2^k = 2^3 = 8$ + 1 (center point)



2^{6-2} Fractional Factorial Design (¼ fractionated)

- **Number of runs (i.e., tests)**
  - 6 controllable factors (k = 6): x_1 thru x_6
  - $n = 2^{k-p} = 2^{6-2} = 16$ runs (about 3 weeks)

- **Response, y_i**
  - n = 16 values

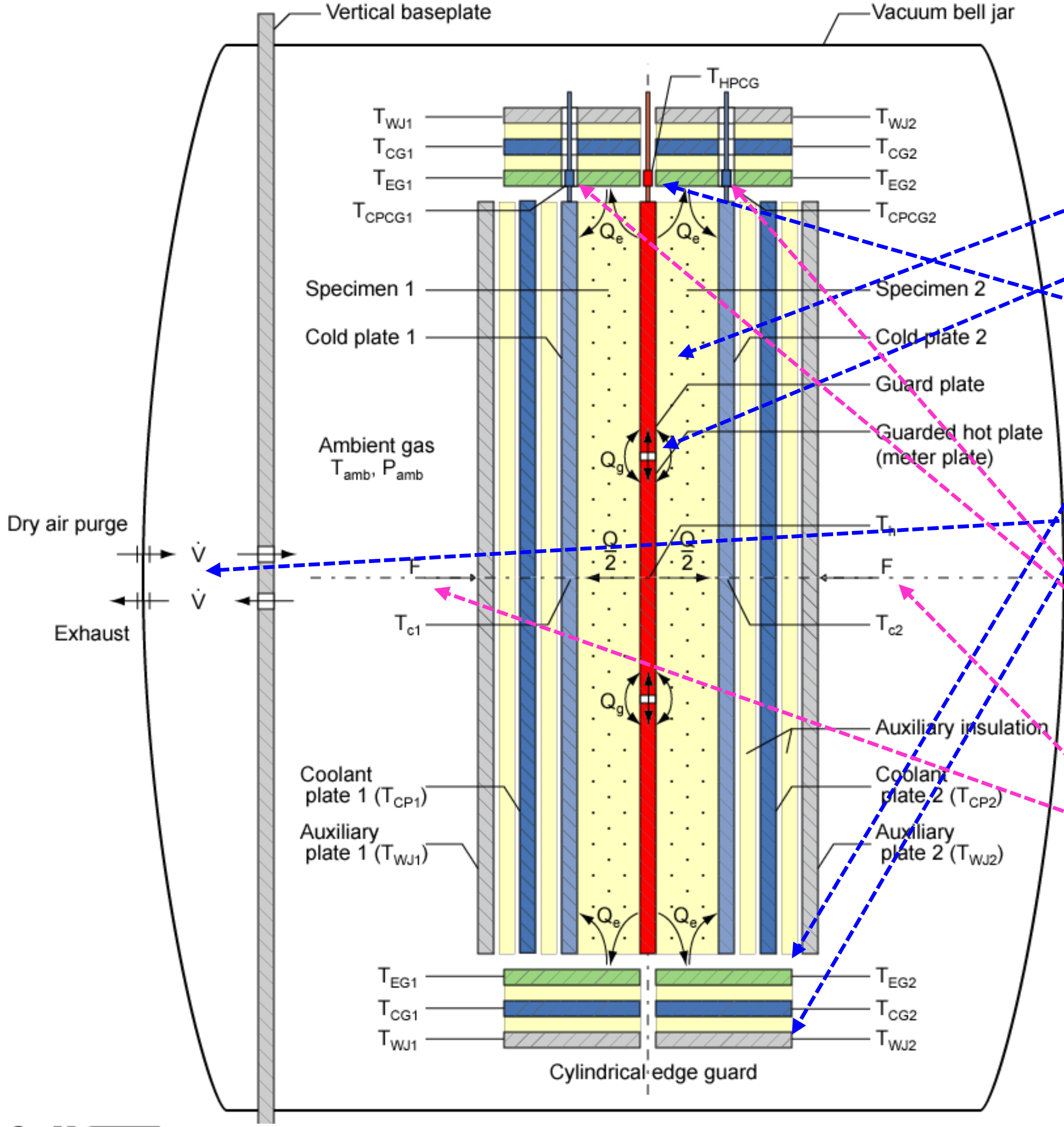
$$y_i = \lambda = \frac{Q\,L_{avg}}{A\,\Delta T_{specimen}}$$

- **Underlying model (2^{6-2})**
  - 6 main effects (β_i)
  - 15 two-term interactions (β_{ij})

$$y = \beta_0 + \frac{1}{2}\begin{bmatrix} \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 + \\ \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{14} x_1 x_4 + \beta_{15} x_1 x_5 + \beta_{16} x_1 x_6 + \\ \beta_{23} x_2 x_3 + \beta_{24} x_2 x_4 + \beta_{25} x_2 x_5 + \beta_{26} x_2 x_6 + \\ \beta_{34} x_3 x_4 + \beta_{35} x_3 x_5 + \beta_{36} x_3 x_6 + \\ \beta_{45} x_4 x_5 + \beta_{46} x_4 x_6 + \\ \beta_{56} x_5 x_6 \end{bmatrix} + e$$

Factor Assignments

"All variables are *not* created equal; some can be varied more easily than others."
G.J. Hahn (1977)



Controlled factors

$x_1 = \Delta T_{specimen}$

$x_2 = \Delta T_{gap}$

$x_3 = \Delta T_{hot\ plate\ conn.\ guard}$

$x_4 = \Delta T_{edge\ guard}$

$x_5 = \Delta T_{water\ jacket}$

$x_6 = \dot{V}$

Fixed factors

$x_7 = \Delta T_{cold\ plate\ conn.\ guard} = 0$ K

$x_8 = T_m = 310$ K

$x_9 = L_{avg} = 26$ mm

$x_{10} = F = 159$ N ➔ 810 Pa

Uncontrolled factors (recorded)

x_{12}: T_{DP}: 205 K to 210 K

x_{13}: T_{amb}: 293.1 K to 294.5 K

x_{14}: P_{amb}: 99.22 kPa to 100.63 kPa

Factors - Coded Settings and Descriptions

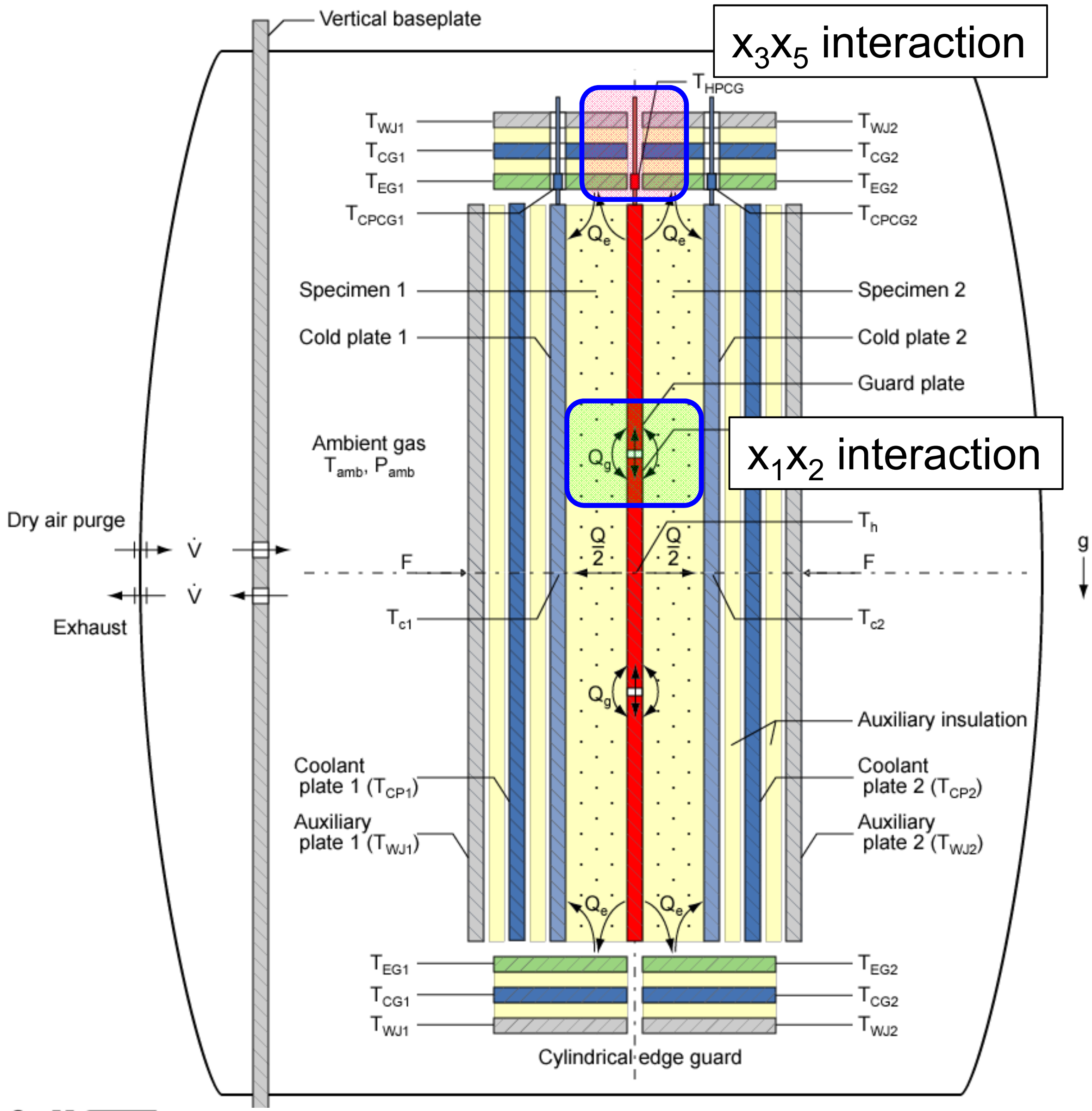
Factor	Coded settings		
	–1 (low)	0 (center)	+1 (high)
x_1, $\Delta T_{specimen}$	20 K	25 K	30 K
x_2, ΔT_{gap}	–0.25 K, guard cooler	0 K	+0.25 K, guard hotter
x_3, $\Delta T_{hot\text{-}plate\ conn.\ guard}$	–0.50 K, guard cooler	0 K	+0.50 K, guard hotter
x_4, $\Delta T_{edge\ guard}$	–2 K, guard cooler	0 K	+2 K, guard hotter
x_5, $\Delta T_{water\ jacket}$	–2 K, guard cooler	0 K	+2 K, guard hotter
x_6, $\dot{V}$ (dry-air purge)	0 m^3/h (Off)	0.7 m^3/h	1.4 m^3/h (Full open)
x_7, $\Delta T_{cold\text{-}plate\ conn.\ guard}$	0 K (fixed)		
x_8, ΔT_{mean}	310 K (fixed)		
x_9, L_{avg}	26 mm (fixed)		
x_{10}, F	159 N (fixed)		

Results (Yates order)

Response, y

Run	x_1 ($\Delta T_{spec.}$)	x_2 (ΔT_{gap})	x_3 (ΔT_{HPCG})	x_4 (ΔT_{EG})	x_5 (ΔT_{WJ})	x_6 ($\dot{V}$)	λ_{exp} W/(m·K)
1	–	–	–	–	–	–	0.03647
2	+	–	–	–	+	–	0.03552
3	–	+	–	–	+	+	0.03075
4	+	+	–	–	–	+	0.03166
5	–	–	+	–	+	+	0.03644
6	+	–	+	–	+	+	0.03549
7	–	+	+	–	–	–	0.03071
8	+	+	+	–	–	–	0.03163
9	–	–	–	+	–	+	0.03648
10	+	–	–	+	+	+	0.03552
11	–	+	–	+	+	–	0.03076
12	+	+	–	+	–	–	0.03166
13	–	–	+	+	+	–	0.03644
14	+	–	+	+	–	–	0.03549
15	–	+	+	+	–	+	0.03073
16	+	+	+	+	+	+	0.03164

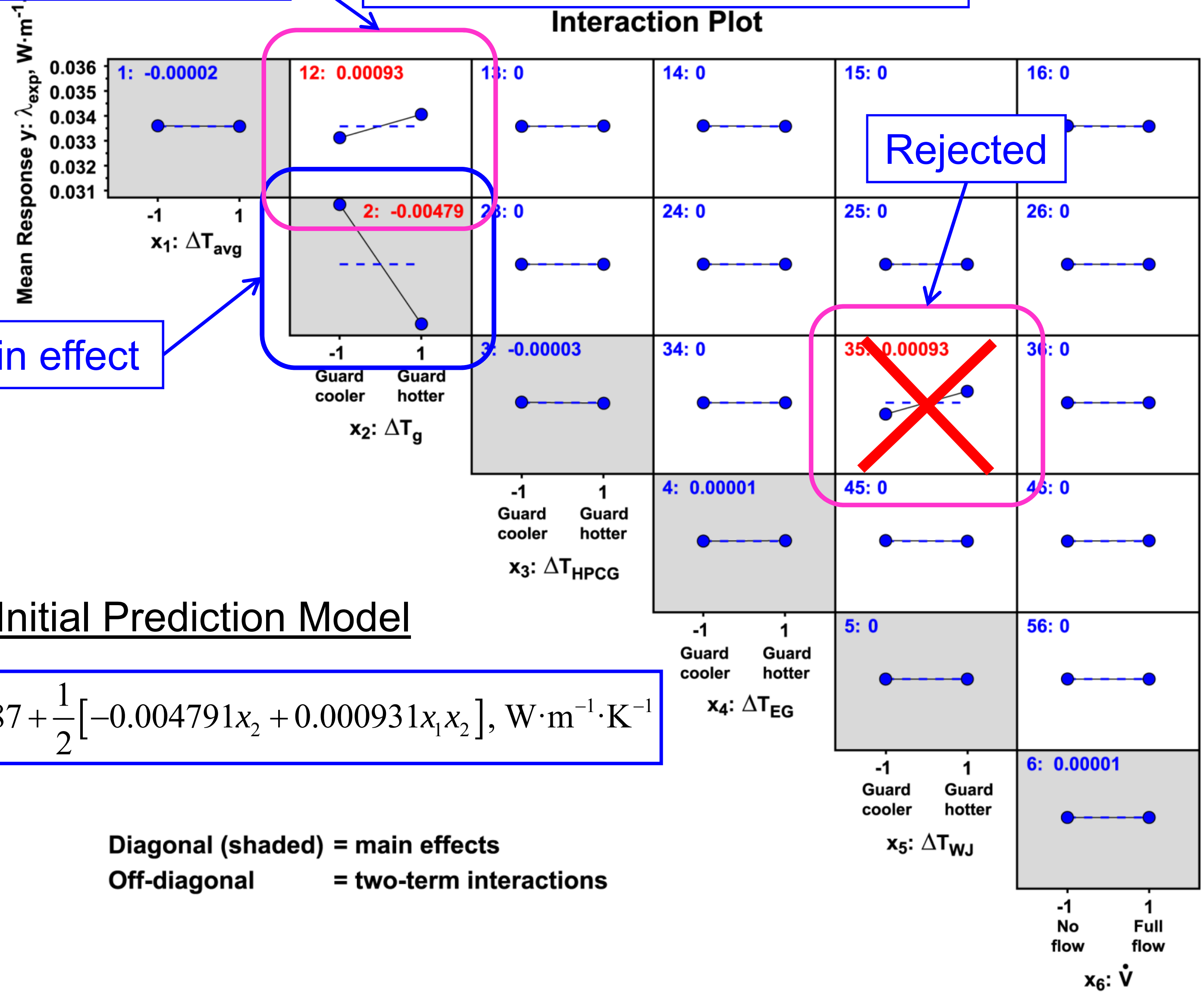
Factor Locations



Main Effects and Interaction Multi-plot

2-term interaction effect

Troussart, *J. Thermal Insulation*, 4 (April 1981)



Main effect

<u>Initial Prediction Model</u>

$$\hat{y} = \lambda_{\text{pred}} = 0.033587 + \frac{1}{2}\left[-0.004791x_2 + 0.000931x_1x_2\right],\ \text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$$

Further Examination of Data

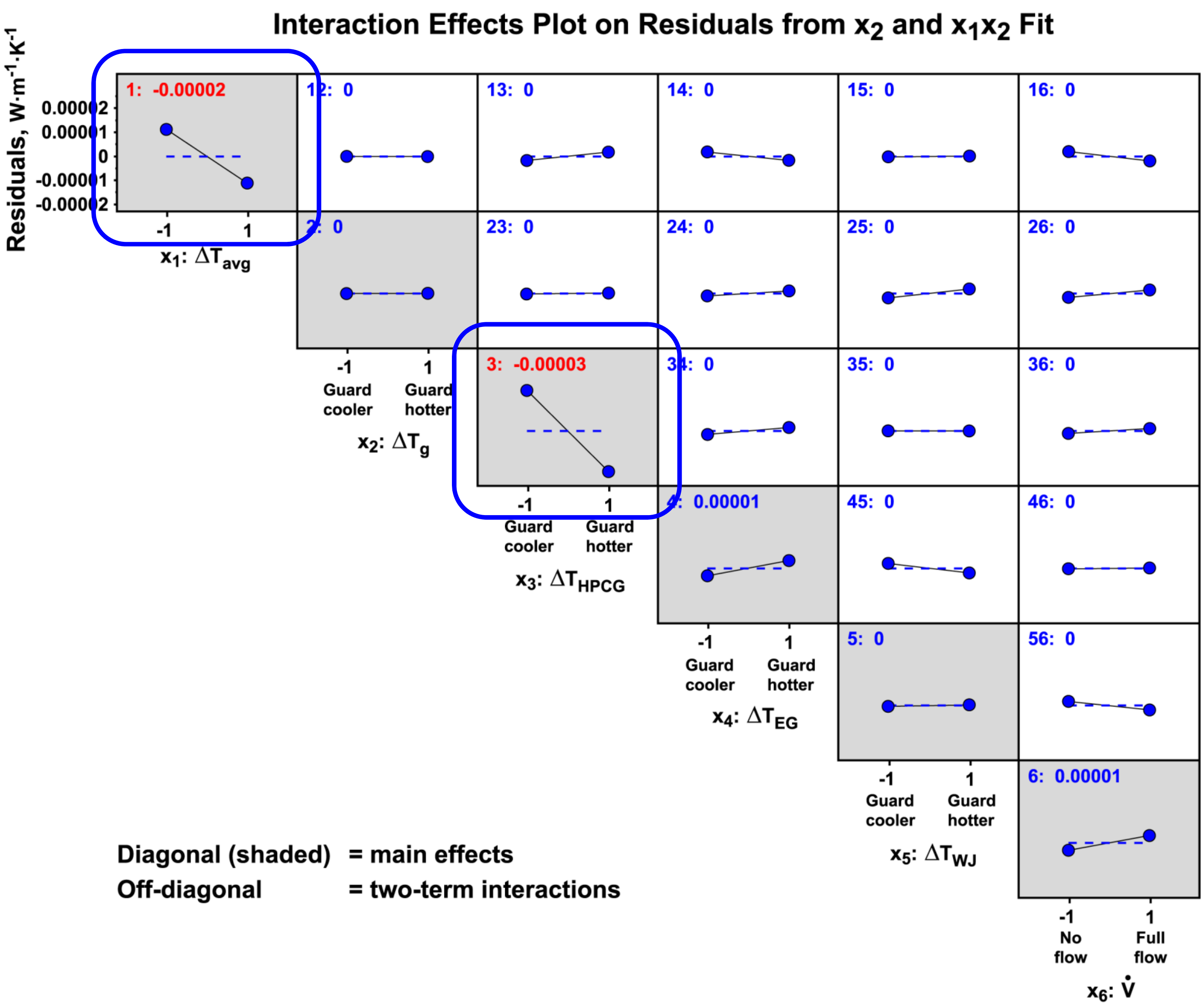
- **The effect of dominant factors in a sensitivity study**
  - Factor x_2: ΔT_g and the interaction term x_1x_2 dominate the other effects by several orders of magnitude
  - When one (or two) factor dominants, it is prudent to check the other factors thoroughly for significance and possible inclusion in the predictive model
  - Method:
    - 1) Yates analysis; and,
    - 2) Graphical interaction plot on the residuals from a model including only the obviously dominant x_2 and x_1x_2 terms
- ***Yates analysis* – least squares fit of an additive model consisting of:**
  - Main effects (6)
  - Appropriate interaction terms

Yates Analysis

Identifier	Estimate ($W \cdot m^{-1} \cdot K^{-1}$)	*t*-value	S_{res} ($W \cdot m^{-1} \cdot K^{-1}$)
Mean	0.033587		0.002520
2	-0.004791	-1377.7	0.000498
12	0.000931	267.9	0.000023
3	-0.000034	-9.7	0.000015
1	-0.000022	-6.4	0.000007
4	0.0000065	1.9	0.0000063
6	0.0000063	1.8	0.0000051
16	-0.0000038	-1.1	0.0000047
13	0.0000036	1	0.0000041
14	-0.0000034	-1	0.0000035
34	0.0000030	0.9	0.0000028
24	0.0000020	0.6	0.0000024
124	-0.0000017	-0.5	0.0000020
134	0.0000015	0.4	0.0000010
5	0.0000006	0.2	0.0000008
23	0.0000004	0.1	0.0000000

Significant

Interaction Effects Plot on Residuals from Fit of x_2 and x_1x_2



Discussion

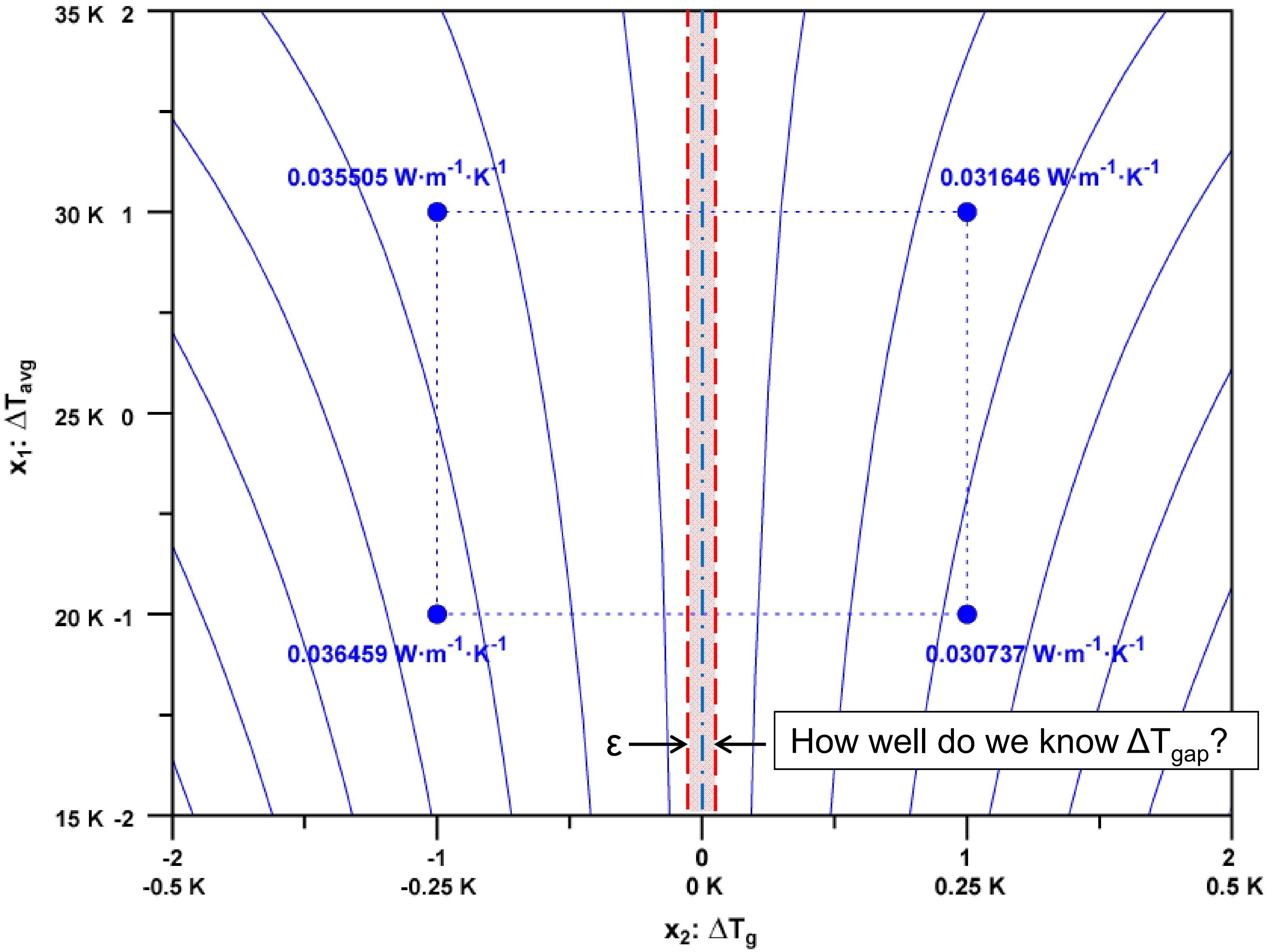
Final Prediction Model

$$\hat{y} = \lambda_{\text{pred}} = 0.033587 + \frac{1}{2}\left[-0.004791x_2 + 0.000931x_1x_2 - 0.000034x_3 - 0.000022x_1\right],\ \text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$$

- where x_i are coded (-1 or +1)

- Guard gap imbalance most important; interaction with ΔT also important
- Localized heat flows can be important ➔ Is active guarding of sensor leads required?
- Suggests that ΔT should be specified in inter-laboratory comparisons (and standard test methods)
- *Example - typical operating conditions:*
  - $x_2 = x_3 = 0$
  - Let $x_1 = -1$ (20 K); $\lambda_{\text{pred}} = 0.033587 + 0.000022 = 0.033609\ \text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$
  - Let $x_1 = +1$ (30 K); $\lambda_{\text{pred}} = 0.033587 - 0.000022 = 0.033565\ \text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$

Contour Plot of Dominant Factors (x_2: ΔT_g and x_1: ΔT_{spec})



Model Predicted Value at Center Point (x_1 = 0, x_2 = 0) = 0.033587 $W \cdot m^{-1} \cdot K^{-1}$

Sensitivity Study Conclusions

- **Factor rank (most important)**
  - 1) Main effect: x_2: (ΔT_{gap})
  - 2) Two-term interaction: $x_1 x_2$: ($\Delta T_{specimen}\ \Delta T_{gap}$)
  - Two other small main effects
    - 3) x_3: ($\Delta T_{\text{hot plate conn. guard}}$) – local heat flow
    - 4) x_1: ($\Delta T_{specimen}$)
- **Results valid over:**
  - Range of the varied factors (x_1 thru x_6)
  - Fixed setting of other factors (x_7 thru x_{11})
    - o T_m = 310 K
    - o L = 26 mm
    - o Material: fibrous-glass board
- **Future work at NIST*:***
  - Additional sensitivity study tests (staged as time permits) are recommended for other temperatures, materials, and thicknesses
  - Publish manuscript in ASTM JOTE