

EMRP SIB 52 Thermo  
**Measurement Uncertainty of High-  
Temperature Guarded Hot Plate Instruments**  
Uncertainty budget workshop 3

18 month meeting  
Stakeholder meeting  
NPL, UK

# Overview

- Measurement uncertainty
  - General remarks
  - ISO GUM JCGM 100:2008
- High-Temperature Guarded Hot Plate
  - Ideal model
  - Real model
  - Uncertainty budget
- Summary

# General remarks

- measurement result:
  - measured quantity value + measurement uncertainty
- uncertainty assessment:
  - to be able to state the measurement uncertainty
  - to analyse the measuring instrument
    - find weak points
    - improve the measuring instrument

- measurement uncertainty
- mathematical model:  $y = f(x_1, x_2, \dots, x_N)$

here (GHP), according to Fourier's eq.

$$\lambda = \frac{\Phi \cdot d}{A \cdot \Delta T}$$

- type A evaluation: statistical analysis
- type B evaluation: other means (e.g. estimate)
- combined standard variance:

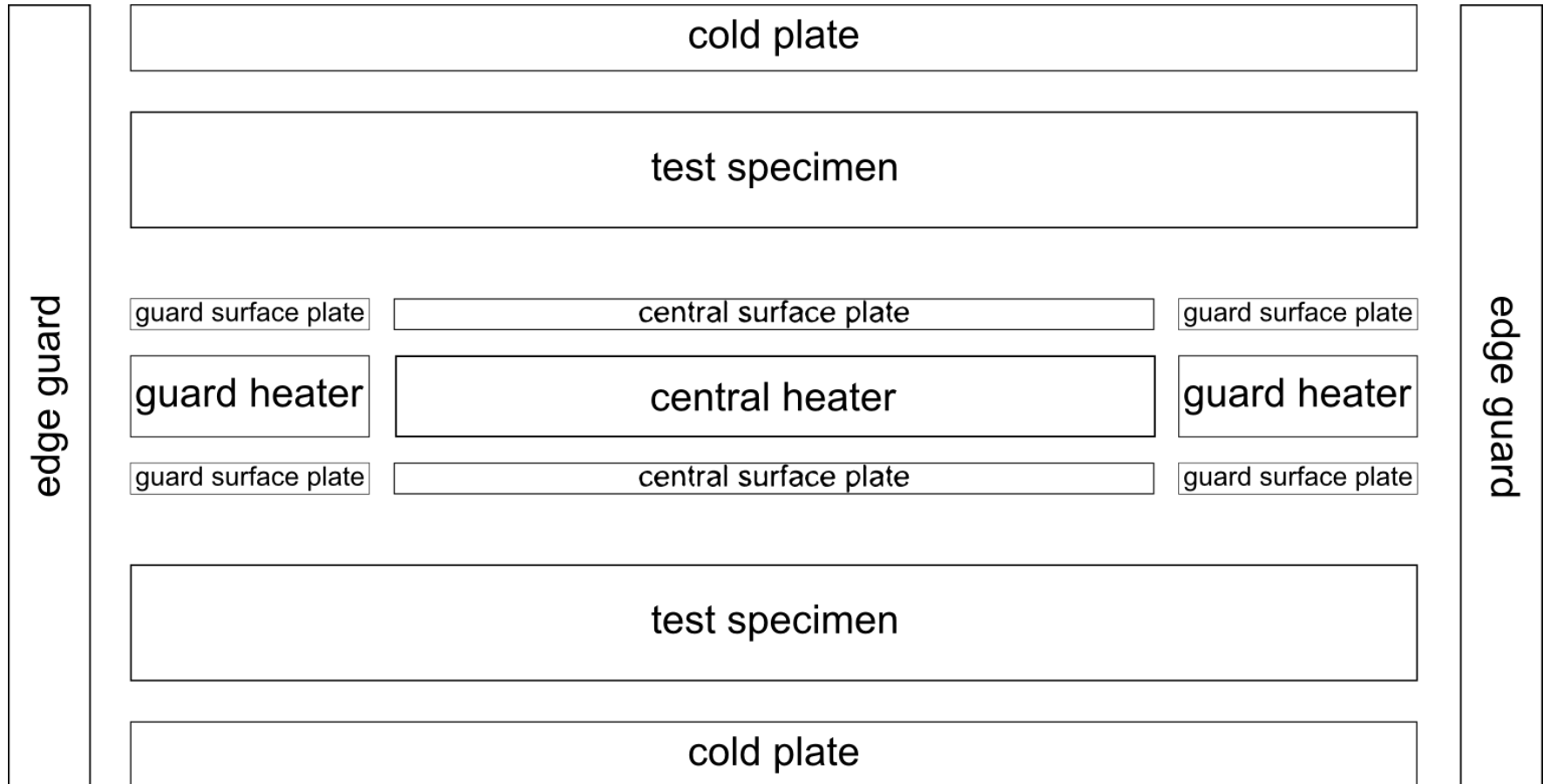
$$u_c^2(y) = \sum_{i=1}^N \left( \frac{\partial f}{\partial x_i} \right)^2 u^2(x_i)$$

- combined standard uncertainty:

$$u_c(y) = \sqrt{u_c^2(y)}$$

# HT Guarded Hot Plate

- two specimens version



# HT Guarded Hot Plate

- Ideal model

$$\lambda = \frac{\Phi \cdot d}{A \cdot \Delta T}$$

rate of heat flow  $\Phi$  supplied to the hot plate

specimen thickness  $d$

measuring area  $A$

temperature difference  $\Delta T$

- model for a **perfect** GHP

# HT Guarded Hot Plate

- Real model
  - start with ideal model
  - correct systematic measurement errors
    - thickness
    - metering area
    - temperature drop
    - power
    - miscellaneous factors

# Uncertainty budget

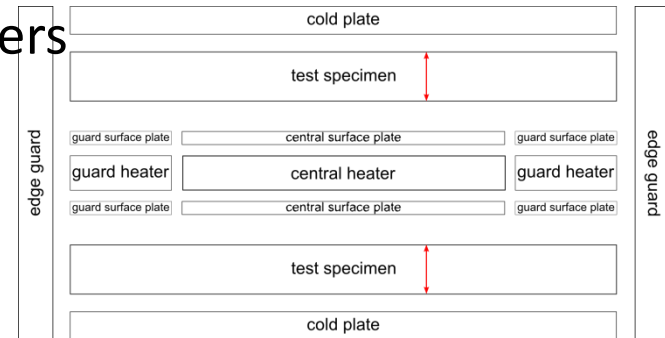
- list of all quantities influencing the measurement uncertainty
  - type A (evaluated by statistical means)
  - type B (evaluated by other means)
- calculation of the standard deviation
- verification
  - certified reference material
    - perform measurements
    - use statistics
      - average and exp. standard deviation of the mean
  - other possibilities to verify measurement uncertainty:
    - intercomparison studies
    - round robin tests



# Thickness measurement

## 1) thickness measurement outside HTGHP apparatus

- resolution and calibration of the callipers
- misalignment
- operator effect
- thermal expansion
  - thermal expansion coefficient
  - uncertainty in mean specimen temperature

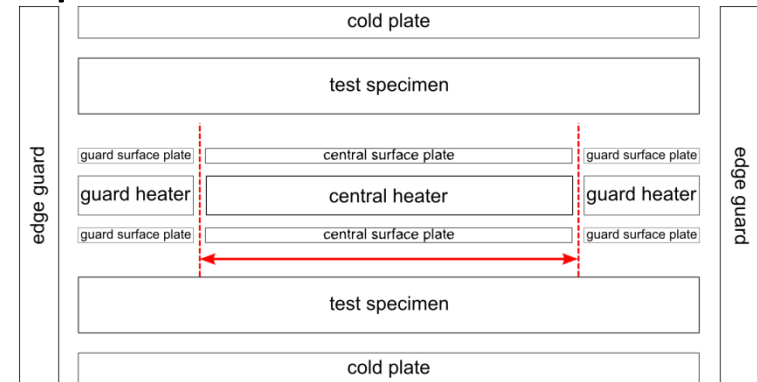


## 2) in-situ thickness measurement

- resolution and calibration of the laser displacement sensor
- correction for thermal expansion of the specimen stack
- variation in specimen thickness over the measuring area
- calibration of straight edge and feeler gauge
- effect of pressure
- compressible samples: thermal expansion of spacer blocks

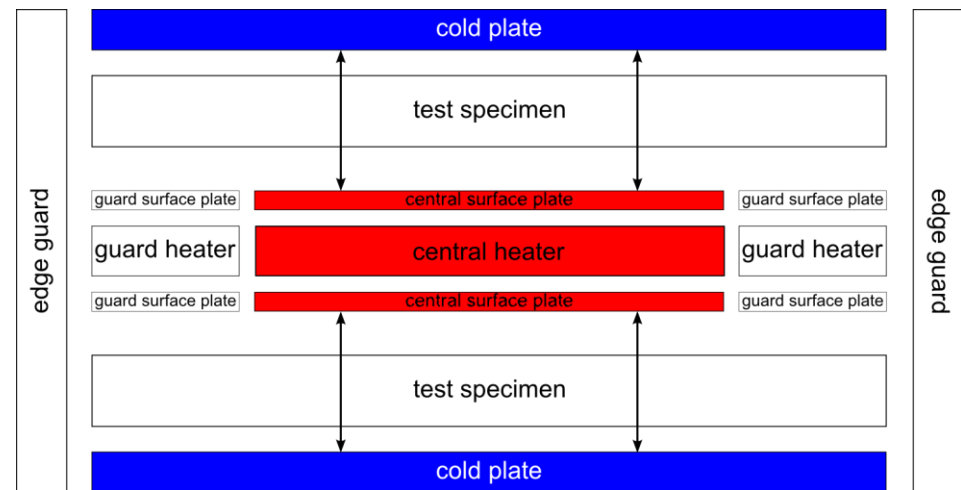
# Metering area

- determination of the metering area:
  - average of the diameter of the hot plate and the diameter of the inner edge of the guard ring
  - correction for thin specimens
- calibration and resolution of the callipers
- misalignment
- operator effect
- thermal expansion
  - thermal expansion coefficient
  - uncertainty in mean hot plate temperature



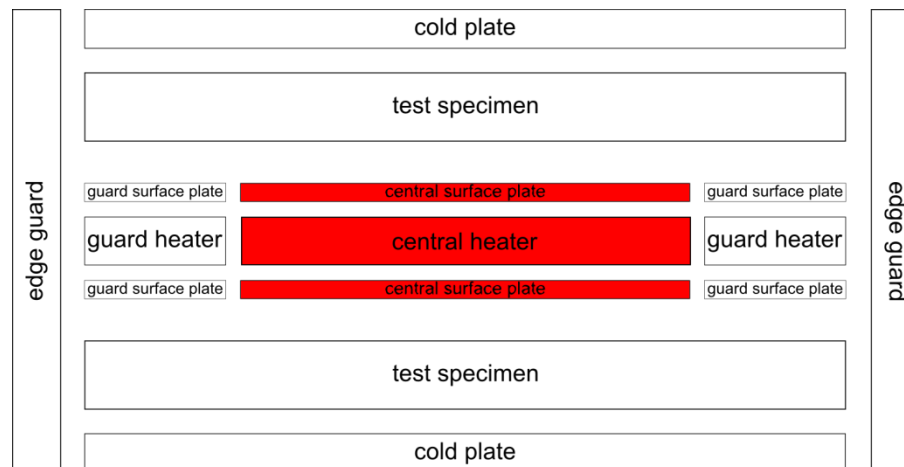
# Temperature drop

- calibration and resolution of the voltmeter
- calibration of temperature sensors
- stability and batch agreement of temperature sensors
- temperature spread
  - across metering area
  - across cold plate
- parasitic voltages  
(connections and switches)
- correction for position of thermocouples



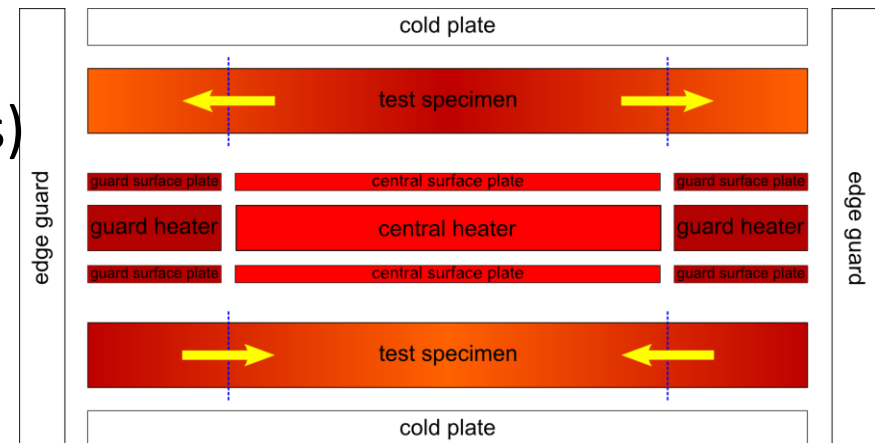
# Power measurement

- calibration of the two voltmeters (voltage and current)
- calibration of the standard resistor
- parasitic voltages
- correction for heated length outside the metering area



# Miscellaneous factors

- temperature imbalance between guard ring and metering area
- temperature imbalance between auxiliary guard and metering area (for single specimen apparatus)
- edge heat loss/gain of the sample
- heat loss/gain between hot plate and auxiliary guard plate (for single specimen apparatus)
- potential deviation from 'true' thermal equilibrium



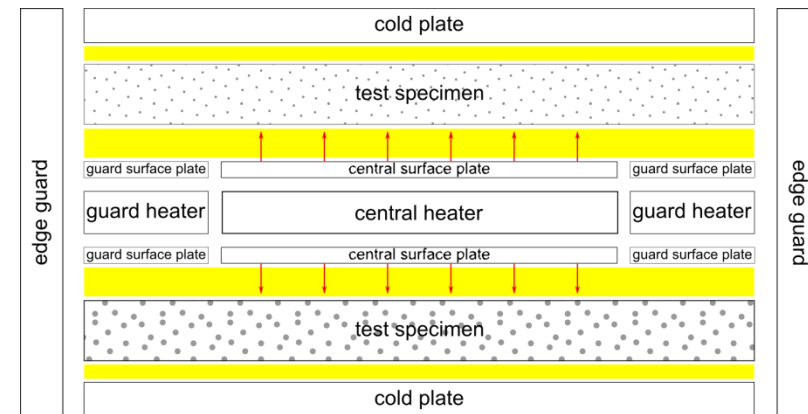
# Absolute mean temperature

(specimen related)

- calibration and homogeneity of the temperature sensors
- in-situ temperature sensor calibration:
  - uncertainty of the PRT
  - calibration and resolution of the voltmeter
  - cold junction temperature

# Miscellaneous factors (specimen related)

- thermal contact resistance between heater plates and sample(s)
- thermal contact resistance between temperature sensors and specimen(s)
- specimen homogeneity
- uncertainty in mean temperature of the sample(s)
- variation of thermal conductivity of the specimen(s) with temperature
- emissivity of the heater plates
- slight difference between the two specimens



# Example uncertainty budget

## Uncertainty budget of PTB's GHP

Quantity $X_i$	Estimate $x_i$	Prob. distrib.	sensitivity coeff. $c_i$	Stand. unc. $u(x_i)$	$u_i(y)$	$[u_i(y)/u_c(y)]^2$
$P_0$	8.9 W	normal	$1.27 \times 10^{-1} \text{ m}^{-1} \cdot \text{K}^{-1}$	$2.3 \times 10^{-4} \text{ W}$	$2.9 \times 10^{-5} \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$	0.007 %
$P_x$	0 W	normal	$-1.27 \times 10^{-1} \text{ m}^{-1} \cdot \text{K}^{-1}$	$3.9 \times 10^{-4} \text{ W}^{-1}$	$-5.0 \times 10^{-5} \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$	0.002 %
$P_{V1}$	0 W	normal	$-1.27 \times 10^{-1} \text{ m}^{-1} \cdot \text{K}^{-1}$	$9.2 \times 10^{-4} \text{ W}^{-1}$	$-1.2 \times 10^{-5} \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$	0.01 %
$P_{V2}$	0 W	normal	$-1.27 \times 10^{-1} \text{ m}^{-1} \cdot \text{K}^{-1}$	$9.2 \times 10^{-4} \text{ W}^{-1}$	$-1.2 \times 10^{-5} \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$	0.01 %
$P_{V3}$	0 W	normal	$-1.27 \times 10^{-1} \text{ m}^{-1} \cdot \text{K}^{-1}$	$9.2 \times 10^{-4} \text{ W}^{-1}$	$-1.2 \times 10^{-5} \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$	0.01 %
$A_0$	$7853.98 \times 10^{-6} \text{ m}^2$	rectangular	$-1.44 \times 10^{-3} \text{ W} \cdot \text{m}^{-3} \cdot \text{K}^{-1}$	$7.9 \times 10^{-6} \text{ m}^2$	$-1.1 \times 10^{-5} \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$	0.9 %
$d_0$	$10 \times 10^{-3} \text{ m}$	rectangular	$1.13 \times 10^{-2} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$	$5.0 \times 10^{-6} \text{ m}$	$5.8 \times 10^{-5} \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$	0.3 %
$\Delta T_0$	10 K	normal	$-1.13 \times 10^{-1} \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-2}$	$5.7 \times 10^{-2} \text{ K}$	$-6.4 \times 10^{-5} \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$	32.9 %
$\Delta T_b$	0 K	normal	$1.13 \times 10^{-1} \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-2}$	$5.7 \times 10^{-2} \text{ K}$	$6.4 \times 10^{-5} \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$	32.9 %
$\Delta T_c$	0 K	normal	$1.13 \times 10^{-1} \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-2}$	$5.7 \times 10^{-2} \text{ K}$	$6.4 \times 10^{-5} \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$	32.9 %
$\lambda$	$1.13 \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$				$0.011 \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$	

published in: U. Hammerschmidt, *Int. J. Thermophys.*, Vol. 23, No. 6, pp. 1551-1570 November 2002



# Summary

- many quantities influence the measurement uncertainty of a HTGHP
- careful uncertainty assessment necessary
  - find weak points
  - improve measurement instrument
  - state measurement uncertainty